

Probabilistic Modeling of Power Systems

Transmission and dispatch

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Working draft

Preface

This book is written for an engineer who knows conservation laws and linear algebra and has never met a probabilistic graphical model. It takes one idea and follows it through: that the structure of a physical system, the way conserved quantities accumulate at places and move between them and cross the system's boundary, is also the structure of every probabilistic question one can ask about that system. A model of the physics is a graph. A model of what we do not know about the physics is the same graph with probability attached to it. Inference on that graph is how a measurement in one place becomes knowledge about another, how a failure is diagnosed, how a decision is justified.

A transmission network is a good system to point that idea at, and in one respect the best. It is the one domain where the idea already has a foothold: operators have estimated the state of their networks from redundant measurements, weighed against the network's own equations, for half a century [82], and the control-room estimator is a small, specialized instance of the construction this book generalizes. Its physics is settled: a power balance at every bus and a flow law on every line. Its instruments are dense by the standards of other physical systems and still leave the questions that matter unanswered. Which reading is wrong? Which breaker status is? How close will tomorrow's peak come to a line's limit after an outage? Its decisions are made under uncertainty at every scale from the next five minutes to the next decade — dispatch, contingency analysis, resource adequacy — on a network whose boundaries are drawn by rule, not by convenience.

Every system worth modeling is a piece cut out of a larger one, trading a conserved quantity with an environment it does not model. For a power network the piece is a balancing area, a region or a substation. The boundary is where one operator's model stops and a neighbor's begins: the tie lines metered for interchange, the generators and loads at the edge of the model, and the lower-voltage network summarized as an injection. That boundary is where the modeler's knowledge is thinnest. As the later chapters show, it is also where the mathematics of inference does its most useful work, because the flows and voltages at a boundary are exactly what the rest of the interconnection needs to know about the inside. Chapter 1 names such a system a *boundary-exchanging system*.

Nothing in the method of Parts I to V is specific to power. The balances, closures and ports are the same whether the conserved quantity is power, charge, mass or heat, and Table 1.3 lines them up. Three companion editions take the same method text through water distribution, district heating and cooling, and data centers. Each is complete in itself, and a reader of this one needs none of them.

Most chapters end twice. First with exercises, which are worked with pencil and paper and whose answers are given for a selection of them. Then with a short list headed *Problems from the studies*. Those are different in kind: each one is a question asked of a real system, stated and solved and committed in the repository beside the study it came from, with a script that

regenerates its answer and a test that checks the answer has not moved. They are there for the reader who wants to run something rather than derive something, and they are the same material the case studies of Part VI are built from, cut down to the size of a problem. The path printed under each is relative to `docs/terra_graph_engine/case_studies/`.

The Introduction says why the book exists, what it inherits from four older traditions, what it adds, and how to read it. Part I is deterministic and establishes the objects. Part II attaches probability to them and states, rather than skips, the places where that attachment needs care. Part III is inference. Part IV asks the questions an operator actually asks. Part V handles time, hierarchy and scale. Part VI is ten complete studies on published test systems, from three lines small enough to enumerate to an annual adequacy study checked against an independent implementation, and each can be reproduced from the material in the earlier parts. Each chapter ends with a short section of notes that says which of its ideas are inherited and from where, and which are the book's own.

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Chapter 0

Introduction

A little after six in the morning, while the load ramps and the generators follow it, the state estimator in a transmission control room flags a residual: one line's measured flow disagrees with what every other measurement implies. The list of explanations is short. The transducer has failed. Or a breaker is recorded as closed when it is open, and the network the estimator is solving is not the network in the field. Or the line's parameters are not what the model says. Or the flow is real, and something on the network has changed. The answer matters within the hour, because the estimate feeds the contingency analysis, and the contingency analysis decides how close to its limits the morning's dispatch may run.

The engineer on shift already has the physics of that network: a power balance at every bus, a flow law on every line, a reference angle at the slack bus and a schedule at every generator. Given the injections, a power-flow solver turns those equations into every angle and every flow. What the solver cannot do is run the other way: take a few hundred readings, one of which is wrong, and say which one, how sure it is, what the flow on the suspect line really is, and whether one more phasor measurement unit would have settled it. Probabilistic graphical models supply that machinery. They are usually taught apart from conservation laws, constitutive relations and engineering solvers, and the engineer who wants both has had to learn two subjects and build the bridge alone. This book is the bridge. Its claim is that the bridge is short, because the graph the probabilistic machinery needs is already there in the equations the engineer wrote, and that once the graph is recognized, every question on the shift's list is a readout of one object. This chapter says why that claim is worth a book, what it inherits from four older traditions, what it adds, and how to read it.

0.1 The questions a grid operator asks

A model of a physical system, as engineering teaches it, is a set of equations and a solver. The equations are the balances, the closures and the boundary conditions; the solver turns known inputs into a computed state. The pipeline is

$$\text{inputs} \longrightarrow \text{solver} \longrightarrow \text{outputs}, \tag{0.1}$$

and it answers one question well: given these inputs, what is the state? Most textbooks on physical modeling stop there, and for design that is often enough.

Operation asks different questions. A transmission network is instrumented more densely than most physical systems, and it is still not enough: flows and voltages arrive from SCADA

every few seconds, phasor measurement units cover a fraction of the buses, and every reading passes through a transducer, a channel and a database that can each be wrong. Line parameters come from design data and drift with temperature and age. Loads and renewable output are forecasts. Breaker statuses are telemetered and occasionally misreported. The questions an operator or a transmission planner actually asks are:

- What is the state of the network now, given SCADA and PMU readings and a model I only partly trust?
- Which line's impedance is off, and by how much?
- What is the injection at a bus nobody meters?
- Is this residual a failed meter, a wrong breaker status, a parameter error or a real flow, and on which line?
- How uncertain is each of those answers?
- Where should the next PMU go, and what would it be worth?
- What would happen if I opened that line, redispatched that unit, or moved that phase shifter's tap?
- Which uncertain load or rating drives the risk of an overload after an outage, and is the risk figure I am about to report a floor, a ceiling or a guess?

None of these is a question the pipeline (0.1) can answer. Each asks the model to run in a direction other than inputs to outputs, or to weigh a reading against a law, or to compare explanations, or to say how far its own answer can be trusted. The pipeline has no place to put a reading, no notion of an explanation, and no measure of trust.

The book's starting observation is that the equations do not have to be rewritten to answer these questions. They have to be read differently. A balance says that certain flows sum to zero at a node; a closure says that a flow and two potentials satisfy a relation; a boundary condition says that a quantity has a value. Each is a statement about which combinations of the variables are admissible. Attach to each a statement of how far from admissible the system may be, attach to each uncertain input a statement of what is known about it, attach to each sensor a statement of what it read and how well, and the collection is a probability distribution over the entire state of the system, built from nothing but the physics and the instruments. Every question in the list is then a question about that distribution, and Parts II to IV of this book are about answering them.

0.2 Where the graph comes from

A textbook on probabilistic graphical models begins with random variables and their joint distributions, defines directed and undirected graphs and the factorizations they encode, develops conditional independence, and then teaches inference on the graph. The examples are alarms and burglaries, diseases and symptoms, letters and their noisy transmissions. It is a coherent subject and the machinery is powerful, but an engineer arriving from the physical side asks a question the textbook does not answer: where did the graph come from? For an alarm and a burglary the graph is a modeling decision, drawn from a story about what causes what. For a transmission line, a substation or an interconnected network it is not a decision at all.

For a physical model the graph is the sparsity pattern of the equations. Each balance reads the flows at one node. Each closure reads one flow and the potentials at its ends. Each boundary condition reads one variable. Draw a node for every variable and a node for every equation, join each equation to the variables it reads, and the result is a factor graph, the bipartite graph

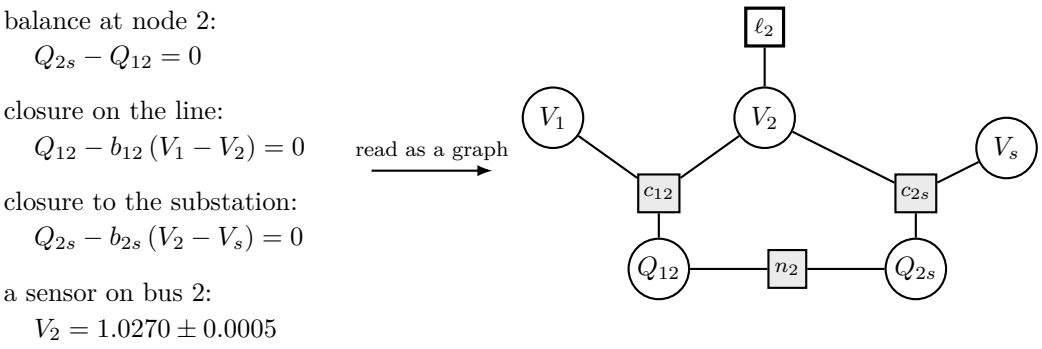


Figure 0.1: The graph comes from the equations. Left: three physics equations of a two-line feeder and one sensor reading, as an engineer writes them. Right: the same four statements as a factor graph. Each equation is a square joined to exactly the variables it reads; the sensor is a square of degree one. Nothing was decided in drawing it.

Table 0.1: What each object of a physical model becomes when probability is attached. The graph is unchanged; only the meaning of the squares is.

In the equations	In the factor graph	With probability attached
balance at a node	a factor on the node's flows	conservation, to within a small width
constitutive relation	a factor on a flow and two potentials	closure, to within its model error
boundary condition	a factor of degree one	a prior on the boundary value
uncertain parameter	a variable in every closure reading it	a prior on the parameter
sensor	(absent)	a likelihood of degree one on its variable
component in or out of service	(absent)	a discrete variable in its closure's scope

on which the probabilistic machinery operates (Figure 0.1). No modeling decision was made in drawing it, because every edge was already an entry of the Jacobian that the solver assembles. The graph is the model's structure, seen.

Attaching probability then changes what the squares mean and not where they are. A balance equation becomes a factor that says the balance holds to within a small tolerance. A closure becomes a factor that says the constitutive law holds to within its model error. An uncertain parameter or boundary condition gets a prior, a square of degree one on its variable. A sensor becomes a likelihood, another square of degree one, on the variable it reads. A component that may be in service or out becomes a discrete variable in the scope of the closure it governs. Table 0.1 lists the correspondence, which Chapter 3 develops. The bridge from the engineer's equations to the graphical model is this table, and it is short.

0.3 One subject, not four

Little in this construction is new in isolation, and it would be wrong to present it as if it were. Four traditions have each built part of it, and the book is written in the conviction that they are one subject that has been taught as four.

Port-based physical modeling. Bond graphs, network thermodynamics and port-Hamiltonian systems supply the domain-independent view of a physical system as conserved quantities at nodes, flows driven by potentials along edges, and conjugate

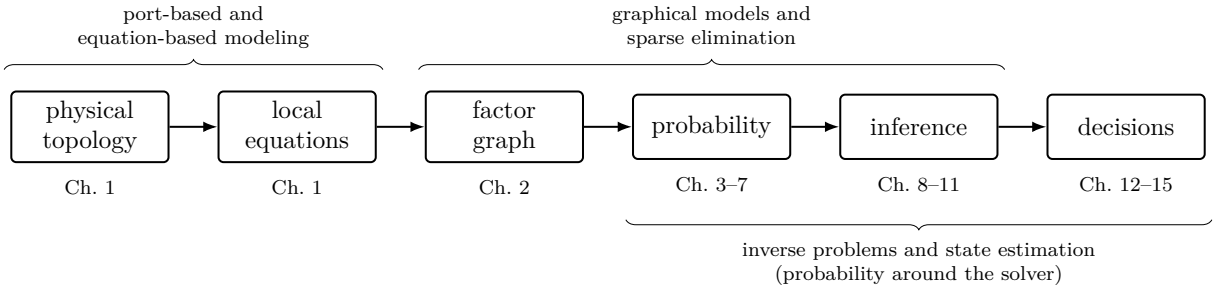


Figure 0.2: The chain of ideas the book follows, with the chapters that cover each link and the traditions that own parts of it. No one tradition carries the object from the topology to the decision.

flow-and-potential pairs at ports, the same structure whether the domain is thermal, fluid, electrical or mechanical [40, 67, 70, 93]. This tradition stops at the equations: it says how to write $\mathbf{F}(\mathbf{z}) = \mathbf{0}$ for any physical system, and then solves.

Equation-based modeling. Acausal modeling languages and the structural analysis of differential-algebraic systems establish that a physical law is a relation among variables, not an assignment of an output from inputs, and they give the bipartite equation-variable structure and its matching theory [5, 20, 64, 69]. This tradition, too, stops at the solve, and it does not attach probability.

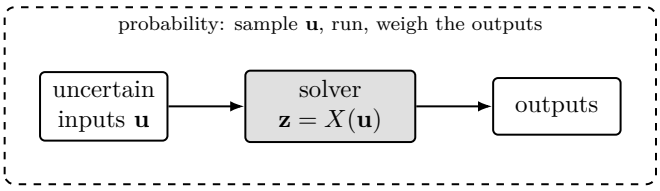
Probabilistic graphical models and sparse elimination. Factor graphs, Markov properties, separators, elimination, fill and treewidth come from the graphical-model and sparse-matrix literatures [16, 46, 48, 50, 78]. This tradition starts from a factorized distribution and asks how to compute with it; it does not say where the factorization comes from, and its graphs are drawn, not derived.

Bayesian inverse problems and state estimation. Priors on uncertain physical inputs, sensors as likelihoods, weighted least-squares estimation under conservation constraints, and the diagnosis of bad data are established in inverse problems, in power-system state estimation, which has run in transmission control rooms since Schweppe and Wildes formulated it [82], and in process data reconciliation [1, 38, 58, 65, 87]. This tradition does attach probability to physics, but usually around the solver: the physics is a black-box map from inputs to observables, and the posterior is over the inputs alone.

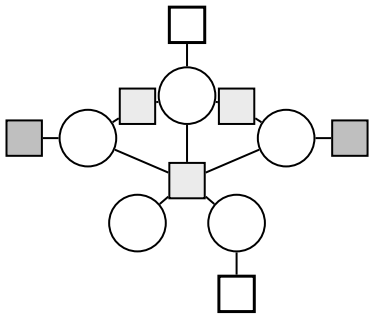
Put the four end to end and they make one chain,

$$\boxed{\text{physical topology} \;\rightarrow\; \text{local equations} \;\rightarrow\; \text{factorization} \;\rightarrow\; \text{probability} \;\rightarrow\; \text{inference} \;\rightarrow\; \text{engineering decisions}} \tag{0.2}$$

of which each tradition owns one or two links and none carries the whole. Figure 0.2 draws it, with the chapters of this book placed along it. The book's purpose is to make the chain feel like one subject: to start where the engineer starts, at the topology and the equations, and to arrive where the operator needs to be, at a decision with its uncertainty stated, without changing the object along the way.



(a) probability around the solver



(b) probability in the structure:
priors (dark) and sensors (open) added to the physics graph

Figure 0.3: Two ways to combine physics and probability. (a) The solver is a black box; probability lives outside it, over its inputs and outputs. (b) The physics equations are the factors; priors and sensors are squares added to the same graph. The book takes the second route and says why.

0.4 Probability in the structure, not around the solver

There is an obvious way to combine a physical model with probability, and most of the fourth tradition uses it: leave the solver as it is, treat it as a function from uncertain inputs to observable outputs, draw inputs from their priors, run the solver, and reason about the outputs. The physics is a black box inside a probabilistic wrapper. Figure 0.3 draws it on the left. It is correct, it is general, and it is how a great deal of uncertainty quantification is done.

The book takes the other route, drawn on the right: the equations themselves are the factors, and probability lives inside the structure, on every variable the physics has. The thesis of the book, in one sentence, is

> Do not put probability around the solver. Put it into the structure of the physical model. (0.3)

Three consequences follow, and they are what the later chapters are about.

There is one object. The deterministic solve is the special case of the probabilistic model in which the priors and sensors are absent and the physics is exact; the two share every row and every Jacobian (Chapter 6 proves this). Adding a sensor adds a row. Adding a question adds nothing: the estimate, the diagnosis, the counterfactual and the risk are readouts of one posterior (Chapter 12).

Structure is preserved, and structure is what computation costs. The wrapper's map from inputs to outputs is dense: every output depends on every input. The physics graph is sparse: every equation reads a few variables. The posterior built in the structure has the sparsity of the physics, and every computation in Part III is affordable because of it. The wrapper's route discards the sparsity the moment it closes the box.

Violation can be seen. A wrapper enforces the physics exactly, because the solver does. When the sensors report something no state of the physics can produce, the wrapper has no answer. The structural model gives each law a width, lets it be violated at a cost, and reports the violation as a diagnosis: the failed transducer, the breaker whose recorded status is wrong, the line whose parameters have drifted (Chapters 4 and 12).

0.5 The boundary

Every system worth modeling is a piece cut out of a larger one. A balancing area trades power with its neighbors, a substation trades power with the distribution feeders below it, a wind farm trades power with the collector system. The piece has an inside that the modeler describes and a boundary across which conserved quantities pass to an outside the modeler does not describe. Chapter 1 calls such a piece a *boundary-exchanging system*, and the book returns to the boundary at every level for a reason that is easiest to state in the language of the third tradition.

In a graphical-model text, a *separator* is a set of variables whose knowledge makes two parts of the graph independent, and finding one is a graph-theoretic exercise whose reward is a cheaper computation. Interconnected power systems drew their separators before any theory asked them to. Balancing areas are joined by tie lines, the tie lines are metered for interchange accounting, and each area's control error is computed from exactly those meters and the frequency. The flows on the tie lines and the voltages and angles at their ends, a flow and its conjugate potential at each port, are exactly the variables that separate an area's interior from the rest of the interconnection (Chapter 5 proves it, and proves that nothing smaller does). So the instruction "find a separator" becomes "cut the interconnection at the tie lines", which is where the interconnection's own rules have already cut it, and the mathematics then explains why that cut is also the cheap place to divide the computation (§8.6 works two regions and one tie line).

This gives one decomposition three readings at once. Physically, a subsystem is summarized to its neighbors by what crosses its ports, exactly, because conservation says the cut flows equal the net source inside. Probabilistically, the ports are a separator, so a subsystem's message to the rest of the system is a function of the ports alone. Computationally, eliminating the interior onto the ports is the Schur complement of sparse elimination, the Kron reduction of network theory, and the region message of a junction tree, which are one operation (Chapter 8). A region can be an equivalent network with error bars, computed once and reused; a large system can be a tree of regions (Chapter 17); and a partition can be judged by its ports (Chapter 18). The three-way connection is the part of the book for which the author has found the least direct precedent, and it is the reason the boundary is in the title.

0.6 From "what is the state" to "what can I know"

The pipeline (0.1) has a direction: inputs in, outputs out. A physical law does not. The relation

$$Q - b\,(V_{\text{tail}} - V_{\text{head}}) \;=\; 0 \tag{0.4}$$

says that four quantities satisfy one constraint. It does not say that Q is computed from the voltages; given Q and V_{tail} it determines V_{head}, and given all three it determines b, which is how a line whose parameters have drifted is found (§12.8). Which variable is unknown is a property of the question, not of the physics (Chapter 2 makes this precise, and shows that for a looped system no fixed direction exists at all). The book keeps the relations as relations, which is what the second tradition insists on and what the first tradition's bond graphs draw, and it is why the factor graph rather than the directed graph is the book's language.

Once uncertainty and observations enter, the absence of direction becomes the point. A reading of one voltage is evidence about everything the physics ties to it. On the export feeder that runs through the book, a single voltage sensor one bus down cuts the uncertainty in the unmeasured reactive injection by a factor of three and a half, and predicts the unmeasured

Table 0.2: The closest prior work, by what it covers. A check marks a substantial treatment; a word marks a partial or specialized one.

	physics graph	equations as factors	probability	general domains	ports as separators	broad inference
bond graphs, port-Hamiltonian [70, 93]	✓	—	—	✓	✓	—
multibody factor graphs [10]	✓	✓	limited	multibody	—	solve, estimate
bond-graph Bayesian networks [55]	✓	indirect	✓	general idea	—	fault diagnosis
factor-graph state estimation [14]	power grid	specialized	✓	power only	regional	state estimation
graphical-model texts [46, 50]	—	—	✓	abstract	graph-theoretic	✓
inverse-problem texts [38, 87]	equations	not generally	✓	✓	—	inverse only
this book	✓	✓	✓	✓	✓	✓

generator-bus voltage to within about a thousandth of a per unit, before any second sensor is read (Chapter 3). That prediction is a *virtual sensor*, and its error bar is part of the answer. The question the model answers has changed from "what is the state, given the inputs" to "what can I know, given what I have", and the second question is the operator's.

0.7 What is inherited and what is the book's

A book that combines old ingredients owes its reader a clear account of which is which. Each chapter closes with a section of notes that does this for its own contents. The account for the whole is short.

Inherited: the port-based view of physics; the acausal reading of its equations; the factor graph, its Markov properties, elimination and treewidth; the Gaussian machinery of least squares, sparse Cholesky and the Laplace approximation; Bayesian inverse problems; state estimation and data reconciliation with their bad-data statistics; the causal calculus of interventions; variance-based attribution; reliability evaluation and contingency analysis. None of these is the book's, and the notes say whose each is.

The book's: the synthesis. Taking an equation-based, conservation-structured physical model and turning its residual structure, unchanged, into the factorization on which inference, diagnosis, uncertainty propagation, attribution and decision queries are performed; the identification of physical ports with probabilistic separators and computational cuts; the intervention semantics for acausal equations; and the insistence, carried through every chapter, that the deterministic model is a corner of the probabilistic one.

The author has looked for this treatment and not found it. Pieces of it exist in several communities that have discovered the connection independently. Multibody dynamics has been written as a factor graph whose sparse structure serves both simulation and estimation, with forward and inverse dynamics as different queries on one graph [10], which is the "relations, not assignments" reading in a single domain. Bond graphs have been converted into Bayesian networks for fault diagnosis [55], which attaches probability to a port-based physical model but does so by assigning the causal directions this book declines to assign. Power systems have been estimated by message passing on a factor graph built from the power-flow equations [14], which is Chapters 6 to 12 in one domain and one query. Table 0.2 sets these beside the four traditions and beside this book. What the table shows is not that any column is empty but that no prior row fills them all, and that several communities have arrived at parts of one construction without a text that says they are parts of one construction. That is the gap the book is written to fill, and "not found" is the strongest claim the author is willing to make for it.

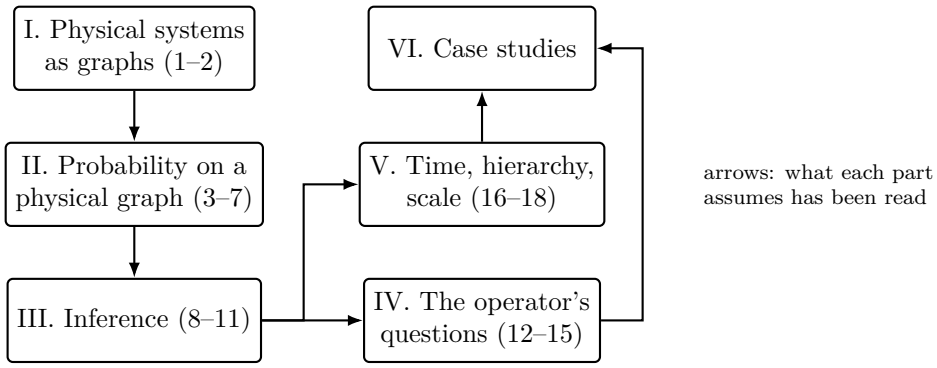


Figure 0.4: The parts of the book and their dependencies. Parts IV and V each need Part III and not each other; Part VI needs both.

0.8 How to read this book

Who it is for. The reader is an engineer who knows conservation laws and linear algebra and has never met a probabilistic graphical model. Nothing about probability beyond the meaning of a density is assumed; Chapter 3 supplies what is needed in a page and Appendix B the Gaussian identities. A reader who knows graphical models and wants the physics will find Chapters 1, 2 and 5 the ones to read first, since they say what the graph is and why the ports separate. The examples are power-system examples; the constructions are not, and a reader who works on heat, water or any other conserved quantity will find nothing in Parts I to V that needs changing but the names.

The parts. Figure 0.4 shows the six parts and what each depends on. Part I is deterministic and builds the objects. Part II attaches probability and states, rather than skips, the places where that attachment needs care. Part III organizes the computation. Part IV asks the operator's questions. Part V handles time, hierarchy and scale. Part VI is a set of complete studies, each reproducible from the material in the earlier parts.

The running examples. Two small systems appear in almost every chapter: an export feeder, a generator that reaches an intermediate bus and then a substation busbar, and a triangle of three buses joined by three lines under the DC approximation of power flow. Both are introduced in Chapter 1, both are small enough to solve by hand, and both acquire priors, sensors, a failed component, a second region, an intervention and a failure set as the chapters go on. The reader who follows them through will have seen every construction of the book on a system whose every number can be checked. Larger systems appear where the small ones cannot show the point: a six-bus network for decomposition and failure, two interconnected areas for common cause, a mesh of buses for cost at scale, and the case studies of Part VI.

The numbers. Every number in every worked example is produced by a short script that accompanies the book, one per chapter, and the text says which. The scripts are a few hundred lines each and use nothing beyond a numerical library; they are meant to be read and rerun, and several exercises ask for exactly that. The case studies of Part VI read their numbers from committed result files of the studies they describe. No number in the book was typed by hand.

Conventions. Each chapter ends with a section on what is in practice, notes on what is inherited and from where, a summary, five exercises, and worked solutions to two of them. Load-bearing conclusions are set apart as numbered principles; there are about three per chapter, and a reader who remembers only those will have the book's argument. Propositions are proved where they are stated. Notation is collected in Appendix A, and the reference implementation that the author used to run every script is mapped to the book's constructions in Appendix C; nothing in the body depends on it.

0.9 What this book does not do

It does not teach physical modeling; it assumes the reader can write a balance and a closure, and it uses the simplest ones. It does not teach probability or graphical models in generality; it teaches the parts a physical model needs, and points to the texts that teach the rest. It does not develop machine learning: no factor in this book is learned from data, though Chapter 12 estimates parameters and Chapter 14 fits a surrogate, and the notes say where the learned versions of its constructions live. It does not treat dynamics faster than a dispatch interval: rotor angles, transient stability, electromagnetic transients and protection timing are absent, and time enters in Chapter 16 only as slow drift, conductor heating and cascading outages.It does not treat continuous space: every system here is a graph of lumped elements, and a field discretized on a mesh appears only as a large graph whose treewidth is a warning. And it does not claim that the constructions it teaches are new; it claims that they are one subject, and it tries to teach them as one.

Notes and further reading

The five gaps that §0.1 to §0.6 describe were articulated in a review of the manuscript, and the positioning of §0.7 follows a literature search made in response to it. The closest works found, Blanco-Claraco, Leanza and Reina's factor-graph multibody dynamics [10], Lo, Wong and Rad's bond-graph Bayesian networks [55], and Chavali and Nehorai's factor-graph state estimation [14], are each discussed in the chapter whose subject they touch. The four traditions of §0.3 are cited there and again in the notes of the chapters that inherit from them; the reader who wants one text per tradition is directed to Karnopp and Rosenberg [40], Benveniste, Caillaud and Malandain [5], Koller and Friedman [46], and Kaipio and Somersalo [38].

Part I

Physical systems as graphs

Chapter 1

Conservation, closure, and the boundary

Every physical system a modeler cares about is a piece cut out of a larger one. A transmission network trades power with the networks around it. A substation trades reactive power with the feeders it supplies. A building trades heat with the weather. The piece has an inside, which the modeler describes in detail, and a boundary, across which conserved quantities pass to an outside the modeler does not describe at all.

This chapter builds the deterministic model of such a piece. It has three ingredients and no probability. The three ingredients are conservation at places, closure along paths, and exchange at the boundary. The reason to build the deterministic model first, and to build it carefully, is that the probabilistic model of Part II inherits its structure entirely: the places, paths and boundary of this chapter become the nodes, factors and separators of the graphical model. Nothing is added to the structure later. Only uncertainty is.

1.1 Three questions about any system

Take any conserved quantity: energy, mass, electric charge, a chemical species. Three questions describe how it lives in a system.

1. Where can it accumulate?
2. How does it move from one place to another?
3. What crosses the boundary?

The first question identifies the *places*. Each place has an amount of the quantity, which may change in time. The amount is an *extensive* state: the energy in a wall, the mass of water in a tank, the charge on a capacitor. Doubling the place doubles the state.

The second question identifies the *paths*. Along a path the quantity flows at some rate: watts, kilograms per second, amperes. What drives the flow is a difference in an *intensive* quantity between the two ends of the path: temperature, pressure, voltage. Doubling the place does not double the intensive quantity; it is a property of the state at a point, not of the amount.

The third question identifies the *boundary*. Some paths lead to places the model does not describe. The quantity that crosses those paths enters or leaves the system. The places on the far side are held at imposed conditions, either an imposed intensive value (the ambient temperature, the grid voltage) or an imposed flow (a heat load, a scheduled injection).

Table 1.1: Four conservation domains and their three quantities.

Domain	Extensive state	Flow	Intensive driver
Energy (thermal)	internal energy E [J]	heat rate Q [W]	temperature T [K]
Mass	mass M [kg]	mass rate $\dot{m}$ [kg/s]	pressure p [Pa]
Charge	charge q [C]	current I [A]	voltage V [V]
Power (DC network)	—	line flow F [W]	angle θ [rad]

Table 1.1 lists the four conservation domains that recur in this book with their extensive state, their flow, and the intensive quantity that drives it. The rows are physically different; the columns are the same three questions. That parallel is the foundation of everything that follows. A method that works on the columns works on every row.

The last row is worth a comment. In the DC approximation of a power network the conserved quantity is real power, the flow on a line is the power it carries, and the driver is the voltage angle difference between its ends. There is no extensive state because the approximation is steady: buses do not store power. The row still fits the three questions. The first answer is simply "nowhere".

1.2 Nodes conserve

The three questions define a graph. The places are nodes; the paths are edges. Write $\mathcal{N}$ for the set of nodes, $\mathcal{E}$ for the set of edges, and $\mathcal{D}$ for the set of conservation domains the model tracks. A model with heat and water flowing through the same pipework has $\mathcal{D} = \{\text{energy}, \text{mass}\}$; a DC power network has $\mathcal{D} = \{\text{power}\}$.

1.2.1 The balance law

At each node n and for each domain d in which n can accumulate, the rate of change of the extensive state equals what flows in, minus what flows out, plus what is generated, minus what is lost:

$$\boxed{\frac{\mathrm{d}X_{n,d}}{\mathrm{d}t} \;=\; \sum_{e \in \mathcal{E}(n)} A_{n,e}\, P_{d,e} \;+\; G_{n,d} \;-\; L_{n,d}} \tag{1.1}$$

Here $X_{n,d}$ is the extensive state of domain d at node n, $\mathcal{E}(n)$ is the set of edges incident on n, $P_{d,e}$ is the flow of d along edge e, $G_{n,d}$ is a source at the node and $L_{n,d}$ a loss. The coefficient $A_{n,e}$ is the sign that says whether edge e delivers to or draws from node n; it is defined in §1.2.3.

Equation (1.1) is the whole of physics that a node contributes. It is linear in the flows, always and exactly. It has no parameters. Whether the domain is energy or mass or charge, whether the node is a wall or a tank or a bus, the balance row looks the same. This is not a modeling convenience. It is the reason the graph is the right object: the graph is the conservation skeleton, and every domain that the model tracks uses that same skeleton.

1.2.2 Node roles

Not every node accumulates. Three roles cover the cases.

Balanced. The node has an extensive state that can change. Equation (1.1) holds with its left side live. A wall, a water tank, a battery.

Zero-storage. The node is a junction or a bus. It has no extensive state, so equation (1.1) holds with $\mathrm{d}X_{n,d}/\mathrm{d}t \equiv 0$: what comes in goes out. A pipe tee, an electrical bus, a mixing point.

Reservoir. The node is on the far side of the boundary. Its intensive state is imposed, not solved. It contributes no balance row, because the model does not claim to know its balance; it appears only as the far end of the edges that cross the boundary. The atmosphere, the utility grid, a feed line.

The first two roles are the inside of the system. The third is the outside. Which nodes are reservoirs is the modeler's first and most consequential decision, and §1.4 returns to it.

A node also declares which domains it is balanced in. A pipe carrying water may need a mass balance at each tee but no energy balance if the pipe is adiabatic; a bus needs a power balance and nothing else. The balance rows exist only where the modeler asks for them. The roles are declared per domain as well: a return manifold whose pressure is held by a pump is a reservoir for mass and yet a balanced node for energy, because the temperature of the water leaving it is the model's to determine. Example 1.3 has such a node.

1.2.3 Incidence

Give every edge a direction, from a tail node to a head node. The direction is a bookkeeping convention, not a physical claim: a flow that runs against the edge's direction is simply negative. The *incidence matrix* $\mathbf{A}$ has one row per non-reservoir node and one column per edge, with entries

$$A_{n,e} = \begin{cases} +1 & \text{edge } e \text{ enters } n \text{ (n is its head)}, \\ -1 & \text{edge } e \text{ leaves } n \text{ (n is its tail)}, \\ 0 & \text{otherwise.} \end{cases} \tag{1.2}$$

Every column of the full incidence matrix (with reservoir rows included) has one $+1$ and one -1, so the columns sum to zero. Dropping the reservoir rows breaks that property for the columns of boundary edges, and that is exactly right: the flow on a boundary edge is not balanced by any row of the model. It is what the system exchanges.

With $\mathbf{A}$ in hand, the sum in equation (1.1) is one row of a matrix product. Stack the flows of domain d into a vector $\mathbf{P}_d$ and the balances into a vector; the balance law for all nodes at once is $\mathrm{d}\mathbf{X}_d/\mathrm{d}t = \mathbf{A}\,\mathbf{P}_d + \mathbf{G}_d - \mathbf{L}_d$.

> **Principle 1.1.** *The incidence matrix is shared by every domain. Energy and mass in the same pipework do not have two graphs. They have one graph, and each domain contributes balance rows on the nodes where it is tracked, using the same* $\mathbf{A}$.

An edge that carries one domain but not another, such as a rotating shaft that transmits energy but no mass, or a conduction path through a solid, simply has no flow variable for the absent domain. Its column contributes zero to that domain's balance rows.

1.3 Edges close

The balance law says that the flows at a node sum to the accumulation. It does not say what any flow is. That is the job of the edge.

END OF THE PREVIEW

Probabilistic Modeling of Power Systems

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