

Capacity Planning and Design by Derivation

Bottlenecks, margins and the cost of uncertainty in the design inputs

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Capacity Planning and Design by Derivation

Volume II

Bottlenecks, input uncertainty, failure and risk

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How to read this book

This book teaches one skill: to take a planner's question stated in the language of the design review, write it as a probability or an expectation over the unknowns of a physical model, derive what that expression requires, compute it, check it, and decide on it. Volume I holds Parts I to III, Chapters 1 to 12: the method itself, the load a data hall can carry (headroom, growth, the design day, diversity), and how much to build (redundancy, unit size, the order date, oversizing). This second volume holds Parts IV to VII, Chapters 13 to 24. It asks where the site's bottleneck is and how much margin the plant really has, what the uncertainty in the design inputs costs and which measurement before committing is worth making, how the design holds when a unit fails, and what the design's expected cost and risk are. Chapter numbers run continuously across the two volumes; a reference to a chapter numbered 12 or below points to Volume I, whose results this volume builds on and does not repeat.

Every chapter follows the progression fixed in Volume I's preface and restated here in one line each: step 0, the plant graph; step 1, the question as a probability or expectation over the state; step 2, the graph extended for the question and the record attached as rows; step 3, the posterior of the state, derived and computed once; step 4, the answers, each question read from that posterior down to its number; step 5, evaluation and checks against the exact posterior and the constructed truth; step 6, the report with its band; step 7, the next measurement, ranked by the width it removes. The five kinds of question and the five computations beneath them are those of Volume I.

The plant of this volume is the site of Volume I's Part III: two data halls on one chilled-water plant with three chillers, one pump, a shared supply pipe, a power feed and a drawn economizer, calibrated on a year of operating records, capacity tests and an outage record. The notation is Volume I's.

The framework the book teaches is physics-constrained probabilistic modeling: a plant written as a graph, its unknowns as a state vector, every reading as a row, and every question as a probability over that state. TERRA is the author's own implementation of this framework; the book names the framework and the computations, not the software, and every number in it is reproducible from the derivations as written.

Each chapter closes with a statement of what to retain, a table of its terms, and exercises at three levels, formulation, derivation and application, each followed by its answer.

Contents

How to read this book **ii**

IV Bottlenecks and Margins **1**

13 Binding constraints **2**
13.1 Which element of the site binds first . . . 2
13.2 The posterior of the state (step 3) . . . 8
13.3 The answers (step 4) . . . 9
13.4 The checks (step 5) . . . 16
13.5 The report (step 6) . . . 17
13.6 The next measurement (step 7) . . . 18
13.7 What to retain . . . 20
13.8 Exercises . . . 21

14 Capacity importance measures **22**
14.1 Which upgrade buys the most headroom . . . 22
14.2 The posterior of the state (step 3) . . . 26
14.3 The answers (step 4) . . . 27
14.4 The checks (step 5) . . . 34
14.5 The report (step 6) . . . 35
14.6 The next measurement (step 7) . . . 36
14.7 What to retain . . . 37
14.8 Exercises . . . 38

15 Shared paths and common-mode limits **40**
15.1 One pipe under two halls . . . 40
15.2 The posterior of the state (step 3) . . . 46
15.3 The answers (step 4) . . . 49
15.4 The checks (step 5) . . . 56
15.5 The report (step 6) . . . 58
15.6 The next measurement (step 7) . . . 59
15.7 What to retain . . . 61
15.8 Exercises . . . 62

16 As-built margin **64**
16.1 Twenty percent on the drawing . . . 64
16.2 The posterior of the state (step 3) . . . 68
16.3 The answers (step 4) . . . 69
16.4 The checks (step 5) . . . 75
16.5 The report (step 6) . . . 77

16.6 The next measurement (step 7) . . . 78
16.7 What to retain . . . 81
16.8 Exercises . . . 82

V The Cost of Input Uncertainty 83

17 Variance budgets for design parameters 84
17.1 How much of the band one handbook guess carries . . . 84
17.2 The posterior of the state (step 3) . . . 89
17.3 The answers (step 4) . . . 90
17.4 The checks (step 5) . . . 98
17.5 The report (step 6) . . . 99
17.6 The next measurement (step 7) . . . 100
17.7 What to retain . . . 102
17.8 Exercises . . . 103

18 Measurements before commitment 105
18.1 Which test narrows the build decision . . . 105
18.2 The posterior of the state and the deciding quantity (step 3) . . . 110
18.3 The answers (step 4) . . . 111
18.4 The checks (step 5) . . . 117
18.5 The report (step 6) . . . 118
18.6 The next measurement (step 7) . . . 120
18.7 What to retain . . . 122
18.8 Exercises . . . 123

19 The value of a pilot 125
19.1 A pilot before the full build . . . 125
19.2 The posterior of the state (step 3) . . . 130
19.3 The answers (step 4) . . . 131
19.4 The checks (step 5) . . . 139
19.5 The report (step 6) . . . 140
19.6 The next measurement (step 7) . . . 141
19.7 What to retain . . . 143
19.8 Exercises . . . 144

VI Design Under Failure 146

20 Headroom with the largest unit out 147
20.1 Hall B with its largest unit out . . . 147
20.2 The posterior of the state (step 3) . . . 152
20.3 The answers (step 4) . . . 153
20.4 The checks (step 5) . . . 162
20.5 The report (step 6) . . . 163
20.6 The next measurement (step 7) . . . 165
20.7 What to retain . . . 167
20.8 Exercises . . . 168

21 Cascading overloads **169**
21.1 A chiller trip on the design day . . . 169
21.2 The posterior of the state (step 3) . . . 175
21.3 The answers (step 4) . . . 176
21.4 The checks (step 5) . . . 185
21.5 The report (step 6) . . . 186
21.6 The next measurement (step 7) . . . 187
21.7 What to retain . . . 189
21.8 Exercises . . . 190

22 Concurrent maintainability **192**
22.1 A maintenance calendar for the site . . . 192
22.2 The posterior of the state (step 3) . . . 198
22.3 The answers (step 4) . . . 199
22.4 The checks (step 5) . . . 206
22.5 The report (step 6) . . . 207
22.6 The next measurement (step 7) . . . 209
22.7 What to retain . . . 211
22.8 Exercises . . . 212

VII Expected Cost and Risk **214**

23 Expected unserved energy **215**
23.1 The site's shed energy over a year . . . 215
23.2 The posterior of the state (step 3) . . . 220
23.3 The answers (step 4) . . . 222
23.4 The checks (step 5) . . . 229
23.5 The report (step 6) . . . 230
23.6 The next measurement (step 7) . . . 231
23.7 What to retain . . . 233
23.8 Exercises . . . 234

24 Risk–cost tradeoffs in design **236**
24.1 The design review's list of additions . . . 236
24.2 The posterior of the state (step 3) . . . 242
24.3 The answers (step 4) . . . 243
24.4 The checks (step 5) . . . 250
24.5 The report (step 6) . . . 251
24.6 The next measurement (step 7) . . . 253
24.7 What to retain . . . 256
24.8 Exercises . . . 257

A Atlas of planning questions **259**

Part IV

Bottlenecks and Margins

Chapter 13

Binding Constraints

13.1 Which element of the site binds first

The site's chilled-water plant has four elements that can stop the load from growing. The three chillers, with the economizer the drawing adds, must carry the heat on the loop; the shared supply pipe must pass the loop's flow within its allowable pressure drop of 150 kPa; the pump must drive that flow within its 12 kW motor; and the power feed must carry the IT load, the chillers and the pump within its 1 600 kVA at power factor 0.95. The design sheet works out, for each, the site IT load at which it reaches its limit on the design day, from nameplate ratings and handbook values. The pipe binds at 1 001.9 kW, the pump at 1 155.2 kW, the feed at 1 202.6 kW and the chillers at 1 228.0 kW. Its order is pipe, pump, feed, chillers, and its recommendation follows from the first entry: the pipe binds first, so upgrade the pipe.

The four numbers come from four different handbooks, a flow coefficient, a pump efficiency, a chiller efficiency and the economizer's conductance, and none of them carries a band. Two of them sit 25 kW apart, the feed and the chillers; the sheet's first and second are 153 kW apart, which looks safe, but only if the pipe's handbook coefficient is right. The question of this chapter is the drill question, verbatim,

Which element binds first as the load grows: the chiller, the pump, the pipe, or the power feed?

It is answered on the site's one fit of Chapter 9, with the binding load of Section 5.1.1 solved for each element in turn. What is new is that the elements are unlike: each binding load comes from its own physics and its own entries of the state, and the answer is not one binding load with a band but an order of four, which is itself a derived quantity of the state, with a probability for each order it can take.

Jargon: binding constraint, binding element, ranking of the elements, order flip, pairwise gap, debottlenecking, allowable pressure drop, flow coefficient, motor rating, apparent power, power factor, auxiliary load, economizer duty, handbook value, Monte Carlo standard error, parameter risk (variance budget)

The *binding constraint* of a plant at a growing load is the limit that is met first, and the element it belongs to is the *binding element*. The *ranking of the elements* is the order of their binding loads, the smallest first; an *order flip* is a state of the plant at which that order differs from the order at the MAP state. The *pairwise gap* of two elements is the difference of their binding loads; its sign says which binds first. *Debottlenecking* is the raising of the binding element's limit so that another element binds; what it buys is the gap to the next element. The *allowable pressure drop* of a pipe is the largest drop its design permits, here 150 kPa across the shared supply pipe. The pipe's *flow coefficient* k_p is the flow, in L/s, that drops one kilopascal; the drop rises with the square of the flow. The *motor rating* of a pump is the shaft power its motor can deliver, here 12 kW. The *apparent power* of a feed, in kVA, is the product of its voltage and current; the real power it can deliver is that times the *power factor*, here $1\,600 \times 0.95 = 1\,520$ kW. The *auxiliary load* is the power drawn by UPS losses and lighting in proportion to the IT load. The *economizer duty* Q_e is the heat the economizer takes from the loop without the chillers' compressors. A *handbook value* is a rating taken from a catalog or a design guide rather than from the plant. The *Monte Carlo standard error* of a probability estimated as a share of N draws is the standard deviation of that share between independent sets of draws. The *parameter risk* of a derived quantity, its *variance budget* in the framework, splits its variance into one share per entry of the state.

Key idea

A binding constraint is a ranking of binding loads, and a ranking is a derived quantity of the state. Each element's binding load is a scenario solved for the load at which its limit is met, with its own gradient and its own band. The element that binds first, and the probability that the order changes, are read jointly from draws of the posterior of the state pushed through every element's binding load. When two elements bind within each other's bands, the order is decided by whatever entry carries the band of their gap, and that entry need not belong to either element's own record.

13.1.1 Binding loads of unlike elements

Chapter 5 solved the scenario node for the load at which a rack inlet meets its limit, and took the hall's binding load as the smallest over its racks. The racks were alike: every inlet rose with the load by the same physics, and they differed only in one fraction. The site's elements are unlike. For each element e write $h_e(\mathbf{z}, Q^*)$ for the quantity its limit constrains at the scenario node, as a function of the free entries $\mathbf{z}$ and of the pinned site IT load Q^*, and $h_e^{\max}$ for the limit. The binding load $Q_e^\dagger$ of element e is the load at which the two meet,

$$h_e(\mathbf{z},\ Q_e^\dagger(\mathbf{z})) = h_e^{\max}, \qquad \mathbf{g}_e = \nabla_{\mathbf{z}} Q_e^\dagger = -\left.\frac{\nabla_{\mathbf{z}} h_e}{\partial h_e / \partial Q^*}\right|_{(\hat{\mathbf{z}},\ Q_e^\dagger(\hat{\mathbf{z}}))}, \tag{13.1}$$

the implicit-function form of equation (5.1), with h_e in the element's own unit: kilowatts of heat for the chillers, kilopascals for the pipe, kilowatts of shaft power for the pump and of real power for the feed. The division by the sensitivity to the load puts every gradient in kilowatts of site IT load, so that the four binding loads stand on one axis and can be compared.

On that axis the plant's binding load is the smallest of the four, and the binding element is the one that attains it. Writing $\pi(\mathbf{z})$ for the ranking, the order of the four elements by their

binding loads at the state $\mathbf{z}$, the smallest first,

$$Q^\dagger(\mathbf{z}) = \min_e Q_e^\dagger(\mathbf{z}), \qquad e^\dagger(\mathbf{z}) = \arg\min_e Q_e^\dagger(\mathbf{z}), \qquad \pi(\mathbf{z}) = \operatorname{argsort}_e Q_e^\dagger(\mathbf{z}), \tag{13.2}$$

three derived quantities of the state: a load, an element and an order. The first has a gradient wherever one element binds alone; the second and the third are constant over regions of the state and jump where two binding loads tie, so they have no gradient at all, and their probabilities are volumes of those regions under the posterior. The order at the MAP state, $\pi(\hat{\mathbf{z}})$, is the one a calibration reports; an order flip is the event $\pi(\mathbf{z}) \neq \pi(\hat{\mathbf{z}})$. For two elements a and b the pairwise gap

$$D_{ab}(\mathbf{z}) = Q_a^\dagger(\mathbf{z}) - Q_b^\dagger(\mathbf{z}), \qquad \mathbf{g}_D = \mathbf{g}_a - \mathbf{g}_b, \tag{13.3}$$

has a gradient wherever both have one, and its sign is the order of the pair: a binds before b exactly when $D_{ab} < 0$. Its variance budget says which entries of the state decide that order.

Principle 13.1. *Each element's binding load is its own scenario solved for the load, carried to the posterior by the implicit-function gradient. The binding element and the ranking have no gradient; they are read as probabilities over draws of the state. The order of a pair is the sign of their gap, and the gap's variance budget names the entries that decide it.*

13.1.2 The site and its four elements (step 0)

The plant is the site of Chapter 9: the two halls on one chilled-water plant, with the three existing chillers C1–C3 of 400 kW nameplate at the design day, the drawn economizer, one pump, the shared supply pipe and the power feed. Its physical relations are equation (9.3), and its state is the eight entries of equation (9.4),

$$\mathbf{z} = (\kappa_{C1},\ \kappa_{C2},\ \kappa_{C3},\ d,\ g_e,\ c,\ l_k,\ l_e)^\mathsf{T},$$

the chillers' as-built log-ratios, their derate per kelvin of wet bulb, the economizer's log-ratio to its handbook conductance, the log of the chillers' efficiency, and the logs of the pipe's flow coefficient and the pump's efficiency. Chapter 9 used the first element alone; this chapter sets all four against each other.

The scenario node. Three inputs are pinned, each a modeling choice. The wet bulb is pinned at the design day's $T_{\text{wb}}^* = 22°\text{C}$: this makes the four binding loads statements about the day the drawing is sized for, and it fails as a statement about any other day. Every unit runs: this is the plant with all three chillers up, and a plant with a unit out, or with the shared pipe out of service, is Chapter 15's question. The plant is the plant as drawn, the three existing chillers and the drawn economizer: this is what the design sheet ranks, and Section 13.4.1 sets the plant as built today, without the economizer, beside it. The site IT load Q^* is free, and the loop's heat is $Q^* + Q_o$ with the halls' other heat $Q_o = 22\,\text{kW}$.

The graph. The site with its four elements is drawn in Figure 13.1. The halls on the left put their heat into the loop, which the pump drives through the shared pipe to the three chillers and the drawn economizer; the feed below carries the IT load, the chillers' power and the pump's. Each of the four elements is drawn with a red border and the limit that binds it, and each is labeled with the entries of the state its binding load depends on. The dashed arrows leave the graph for the operating record of Chapter 9; the economizer has none.

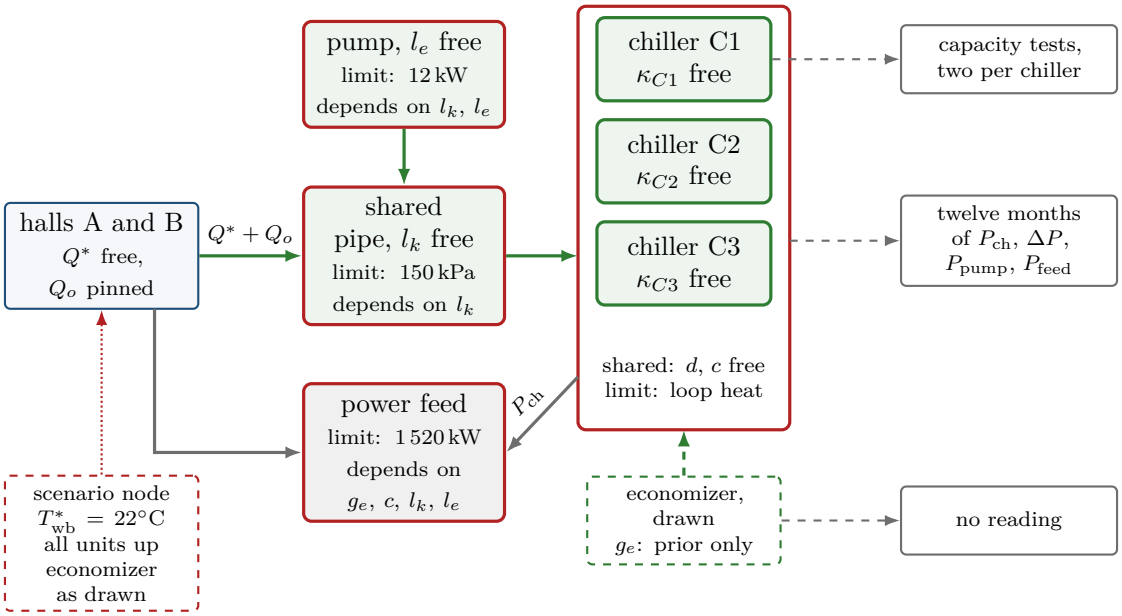


Figure 13.1: The site at the design day with its four elements (red borders), each labeled with its limit and with the entries of the state its binding load depends on. Free (green): the three chillers' as-built log-ratios with the derate d and the efficiency c they share, the pipe's log flow coefficient l_k and the pump's log-efficiency l_e. The economizer (dashed) is drawn but not built; its log-ratio g_e keeps the handbook prior, and no reading sees it. Gray: the power the feed carries, the IT load with its auxiliary load and the chillers' and the pump's power. Observed (dashed arrows): the capacity tests and the twelve monthly windows of Chapter 9. The dotted arrow marks the scenario node (red, dashed), which pins the wet bulb and the units and frees the site IT load Q^*.

Modeling choices at the graph. The plant is taken in steady state at the design day, as in Chapter 9: this makes each limit an algebraic relation in the load, and it fails in the minutes after a load step. The chilled-water supply and the loop's rise are pinned at 14°C and 10 K: this makes the loop's flow proportional to its heat, and it fails if the plant resets the supply or lets the rise grow as the load grows. The pump has one efficiency at every flow and the halls' own pressure drop is pinned at 100 kPa: this keeps the pump to one entry, and it fails where the pump's efficiency falls away from its best point or the halls' valves open as the load grows. The feed's power factor is pinned at 0.95: this makes its limit a real power, and it fails if the power factor falls with the load.

13.1.3 The four binding loads

At the scenario node the loop's flow is $V^* = (Q^* + Q_o)/(\rho c_p\,\Delta T_w)$ in L/s, the pipe's drop is $\Delta P^* = (V^*/k_p)^2$ and the pump's power is $P^*_{\text{pump}} = (\Delta P^* + \Delta P_{\text{h}})\,V^*/(1000\,\eta)$, by equation (9.3). At the design day the derate factor is one, the economizer's duty is $Q_e = 50\,\mathrm{e}^{g_e}$ kW by equation (9.5), and the chillers' efficiency is $\text{COP} = \mathrm{e}^{c}\,[1 - 0.02\,(22 - 18)] = 0.92\,\mathrm{e}^{c}$. Each element's limit, set equal to its value at the load, gives its binding load.

The chillers. The chillers with the economizer carry the loop's heat while $Q^* + Q_o \le \sum_i \text{Cap}_i + Q_e$. The limit is met at

$$Q^{\dagger}_{\text{ch}}(\mathbf{z}) = \sum_i 400\,\mathrm{e}^{\kappa_i} + 50\,\mathrm{e}^{g_e} - Q_o, \qquad \frac{\partial Q^{\dagger}_{\text{ch}}}{\partial \kappa_i} = 400\,\mathrm{e}^{\kappa_i}, \qquad \frac{\partial Q^{\dagger}_{\text{ch}}}{\partial g_e} = 50\,\mathrm{e}^{g_e} = Q_e, \tag{13.4}$$

explicit in the state: the heat side of the limit does not depend on the state, so the sensitivity in equation (13.1) is one. The gradient's entries are each unit's as-built capacity and the economizer's duty, in kW; a log-ratio moves the binding load by the capacity it scales.

The pipe. The pipe's drop meets its allowable $\Delta P_{\max} = 150$ kPa when the flow reaches $k_p\sqrt{\Delta P_{\max}}$, so that

$$Q^{\dagger}_{\text{pipe}}(\mathbf{z}) = \rho c_p\,\Delta T_w\,\sqrt{\Delta P_{\max}}\,\mathrm{e}^{l_k} - Q_o, \qquad \frac{\partial Q^{\dagger}_{\text{pipe}}}{\partial l_k} = Q^{\dagger}_{\text{pipe}} + Q_o, \tag{13.5}$$

the loop's heat at the limit, proportional to the flow coefficient, and every other entry zero. The pipe's binding load is a function of one entry, monotone in it.

The pump. The pump meets its motor rating $P_m = 12$ kW at the flow $V^{\dagger}$ that solves $h(V, \mathbf{z}) = P_m$ with $h = (V^3/k_p^2 + \Delta P_{\text{h}} V)/(1000\,\eta)$, a cubic in the flow with one positive root. Its sensitivities are $\partial h/\partial V = (3\Delta P + \Delta P_{\text{h}})/(1000\,\eta)$, $\partial h/\partial l_k = -2\,\Delta P\,V/(1000\,\eta)$ and $\partial h/\partial l_e = -h = -P_m$ at the root, with ΔP the pipe's drop at $V^{\dagger}$, so equation (13.1), with the load entering through the flow, gives

$$\begin{gathered} Q^{\dagger}_{\text{pump}} = \rho c_p\,\Delta T_w\,V^{\dagger} - Q_o, \\ \frac{\partial Q^{\dagger}_{\text{pump}}}{\partial l_k} = \rho c_p\,\Delta T_w\,\frac{2\,\Delta P\,V^{\dagger}}{3\,\Delta P + \Delta P_{\text{h}}}, \qquad \frac{\partial Q^{\dagger}_{\text{pump}}}{\partial l_e} = \rho c_p\,\Delta T_w\,\frac{(\Delta P + \Delta P_{\text{h}})\,V^{\dagger}}{3\,\Delta P + \Delta P_{\text{h}}}, \end{gathered} \tag{13.6}$$

both positive: a larger pipe or a better pump lets the pump drive more flow before its motor binds. The pump shares the pipe's entry l_k, since the pipe's drop is part of the head the pump supplies.

The feed. The feed meets its real power $S\cos\varphi = 1\,600 \times 0.95 = 1\,520\,\text{kW}$ where

$$F(Q^*, \mathbf{z}) = (1+a)\,Q^* + \frac{Q^* + Q_o - Q_e}{\text{COP}} + P^*_{\text{pump}} - S\cos\varphi = 0,$$

the IT load with its auxiliary fraction $a = 0.06$, the chillers' power for the heat the economizer leaves them, and the pump's power. The sensitivity to the load is $\partial F/\partial Q^* = 1 + a + 1/\text{COP} + \partial P^*_{\text{pump}}/\partial Q^*$, and equation (13.1) gives

$$\begin{aligned} \frac{\partial Q^\dagger_{\text{feed}}}{\partial c} &= \frac{P_{\text{ch}}}{\partial F/\partial Q^*}, \qquad \frac{\partial Q^\dagger_{\text{feed}}}{\partial g_e} = \frac{Q_e/\text{COP}}{\partial F/\partial Q^*}, \\ \frac{\partial Q^\dagger_{\text{feed}}}{\partial l_k} &= \frac{2\,\Delta P\, V/(1000\,\eta)}{\partial F/\partial Q^*}, \qquad \frac{\partial Q^\dagger_{\text{feed}}}{\partial l_e} = \frac{P_{\text{pump}}}{\partial F/\partial Q^*}, \end{aligned} \tag{13.7}$$

every quantity at the binding load. A more efficient chiller plant, a larger economizer, a larger pipe or a better pump each lowers the power drawn at a given load and raises the load at which the feed binds; each entry of the feed's gradient is the power that entry saves, divided by the power one more kilowatt of IT load costs.

The pattern. Read together, equations (13.4) to (13.7) fix which entries each binding load depends on before any number. With $*$ for an entry that can be nonzero, the four gradients have the pattern

$$\begin{bmatrix} \mathbf{g}^{\mathsf{T}}_{\text{ch}} \\ \mathbf{g}^{\mathsf{T}}_{\text{pipe}} \\ \mathbf{g}^{\mathsf{T}}_{\text{pump}} \\ \mathbf{g}^{\mathsf{T}}_{\text{feed}} \end{bmatrix} = \begin{bmatrix} * & * & * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * \end{bmatrix}, \tag{13.8}$$

columns in the order of equation (9.4). Three facts are read here. The derate d has no entry: at the design day its factor is one. The chillers and the pipe share no column, so their gap's gradient $\mathbf{g}_{\text{pipe}} - \mathbf{g}_{\text{ch}}$ has the chillers' entries with their signs reversed and the pipe's entry beside them, and its variance is the sum of the two binding loads' variances and of the covariances between their entries. And the economizer's g_e, which no reading sees, sits in the chillers' row and the feed's: the drawn economizer moves two of the four binding loads.

13.1.4 The planner's questions (step 1)

The planner asks four questions of the site as drawn, and step 1 writes each as a probability or a variance over the state, with the evidence on the right of the bar. Table 13.1 lists them. The first column is the question in the planner's words, the drill question first; the second the general form of its category in the notation of Chapter 2; the third its mathematical form for the site; the fourth its category; the fifth the computation of Section 2.3.6 that answers it; and the last the subsection where it is answered. The drill question and the third are predictions under a threshold over a family of events, the binding element and the order; the second and fourth are estimation, the four binding loads with their bands and the split of the pipe-against-chillers gap. The answers are given in the order two, one, three, four, because the binding element is read from the binding loads.

13.1.5 The evidence as rows (step 2)

Step 2 attaches the operating record of Chapter 9 to the graph, unchanged: two capacity tests per chiller, twelve monthly windows of chiller power, pipe pressure drop, pump power and feed power, and eight prior rows, 62 rows on the 8 entries (Section 9.1.5). The outage record is not used: every unit runs at this scenario node, so the rates of failure and repair do not enter.

Table 13.1: The questions asked of the site at the design day, with all units up and the drawn economizer. $\mathbf{y}$ is the operating record and the priors of Chapter 9; $Q_e^\dagger$, π and D are the binding loads, the ranking and the pairwise gap of equations (13.1) to (13.3).

Planner's question	General form	Mathematical form	Category	Computation	Answered in
Which element binds first as the load grows: the chiller, the pump, the pipe, or the power feed?	$\Pr(f(\mathbf{z}) \geq f^* \mid \mathbf{y})$ over a family	$\Pr(e^\dagger(\mathbf{z}) = j \mid \mathbf{y})$, j the chillers, the pipe, the pump, the feed	prediction under a threshold	posterior	13.3.2
At what load does each bind, and how sure?	$\Pr(f(\mathbf{z}) \leq f^* \mid \mathbf{y})$, quantiles	$Q_e^\dagger$ per element and $Q^\dagger = \min_e Q_e^\dagger$: median and 90 % band	estimation	posterior	13.3.1
What is the probability that the order flips?	$\Pr(f(\mathbf{z}) \geq f^* \mid \mathbf{y})$ over a family	$\Pr(\pi(\mathbf{z}) \neq \pi(\hat{\mathbf{z}}) \mid \mathbf{y})$, and each order's probability	prediction under a threshold	posterior	13.3.3
What decides between the pipe and the chillers?	$\mathrm{Var}(f(\mathbf{z}) \mid \mathbf{y})$, split by entry	the budget of $\mathbf{g}_D^\mathsf{T}\boldsymbol{\Sigma}\mathbf{g}_D$, $D = Q_{\text{pipe}}^\dagger - Q_{\text{ch}}^\dagger$	estimation	posterior	13.3.4

Which row acts on which entry. By the pattern (13.8) each binding load is read through a different part of the record. The pipe's entry l_k is read by every month's pressure drop, twelve rows that see it and nearly nothing else. The pump's l_e is read by the monthly pump power, beside l_k. The feed's entries are read by the monthly chiller and feed power, through the efficiency c. The chillers' log-ratios are read by the six capacity tests. The economizer's g_e is read by nothing: the economizer is not built, no month ran it, and its row is the handbook prior alone, $g_e \sim \mathcal{N}(0,\ 0.35^2)$. Two of the four binding loads, the chillers' and the feed's, carry that prior through their gradients.

Modeling choices at the rows. The rows are those of Chapter 9, with its choices: each reading Gaussian, unbiased and independent given the state, and each prior independent. One choice matters more here than there. The economizer's prior is the whole of what the fit knows of it, so every answer that involves the chillers inherits whatever the handbook got wrong; the record cannot correct it, and Section 13.4.1 shows what the answers become without it.

13.2 The posterior of the state (step 3)

All four questions of Table 13.1 are read from one posterior, the site's one fit with its draws, as Chapter 9 builds them. No row is added and no fit is repeated; this section cites the posterior and the draws the answers use.

13.2.1 The fit of the site, cited

The MAP state $\hat{\mathbf{z}}$ of the eight entries and the Laplace covariance $\boldsymbol{\Sigma} = (\mathbf{J}^\mathsf{T}\mathbf{J})^{-1}$ at $\hat{\mathbf{z}}$ are those of Section 9.2.2, the Gauss–Newton fit of the operating record in the form of equation (4.38), tabulated with the stated truth in Table 9.2. Four of its facts carry this chapter. The chillers' log-ratios are -0.0909, -0.0394 and -0.1157, each to about 0.017, so that the units deliver 365.2, 384.5 and 356.3 kW at the design day. The pipe's l_k is 0.8083 with posterior spread

0.0046, 0.015 of its prior spread, and above the handbook's $\ln 2 = 0.6931$: the pipe passes more flow than the handbook says. The pump's l_e is -0.2891 with spread 0.0190, and the efficiency c is 1.6176 with spread 0.0061. The economizer's g_e stays at its prior, 0 with spread 0.35, uncorrelated with every other entry, since no row sees it.

13.2.2 The draws of the state

The answers read the posterior through the draws of Section 9.2.4: $N = 20\,000$ states $\mathbf{z}_i$ from the Laplace normal $\mathcal{N}(\hat{\mathbf{z}}, \boldsymbol{\Sigma})$, with the seed 9, and their weights w_i of equation (5.16), which reweight them to the exact posterior of the operating record with an effective sample size of 19 913. Each draw is pushed through the four binding loads, the chillers' and the pipe's by equations (13.4) and (13.5), the pump's by solving its cubic for the flow, and the feed's by solving $F = 0$ for the load, which gives one row $(Q^\dagger_{\text{ch}}, Q^\dagger_{\text{pipe}}, Q^\dagger_{\text{pump}}, Q^\dagger_{\text{feed}})_i$ per draw, and with it the binding element and the ranking of that draw. Every answer of step 4 is a function of these rows.

13.3 The answers (step 4)

Every answer is read from the draws of Section 13.2.2. The reported value is the share or the quantile over the equal-weight draws, the joint reading over the four elements; the exact posterior stands beside it with the gap; and the delta form on each element, from its gradient at $\hat{\mathbf{z}}$, stands beside both. The binding loads come first, because the binding element is read from them.

13.3.1 Each element's binding load

Planner's question. At what load does each bind, and how sure? The question asks for the four binding loads, each with its band, and for the plant's binding load, the smallest of them.

General form. An estimation question asks for the posterior distribution of a derived quantity, equation (2.9),

$$\Pr(f(\mathbf{z}) \le f^* \mid \mathbf{y}) = \int \mathbf{1}\big[f(\mathbf{z}) \le f^*\big]\, p(\mathbf{z} \mid \mathbf{y})\, \mathrm{d}\mathbf{z},$$

summarized by its median and its central 90 % credible interval, equation (2.11), between the 5 % and the 95 % points.

Specialized form. The question is equation (2.9) with $f(\mathbf{z}) = Q^\dagger_e(\mathbf{z})$ for each element, by equations (13.4) to (13.7), and with $f(\mathbf{z}) = Q^\dagger(\mathbf{z}) = \min_e Q^\dagger_e(\mathbf{z})$ of equation (13.2) for the plant. The reported quantiles are those of the draws,

$$\hat{F}_e(q) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\big[Q^\dagger_e(\mathbf{z}_i) \le q\big], \qquad \hat{F}(q) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\Big[\min_e Q^\dagger_e(\mathbf{z}_i) \le q\Big], \tag{13.9}$$

and the exact posterior's are those of the same draws under the weights w_i. The delta form of each element is the normal $\mathcal{N}(Q^\dagger_e(\hat{\mathbf{z}}),\ \mathbf{g}_e^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_e)$ with the gradient of equation (13.1), and its band is $Q^\dagger_e(\hat{\mathbf{z}}) \pm 1.645\,(\mathbf{g}_e^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_e)^{1/2}$. For the pipe, a function of the one normal entry l_k and monotone in it, the quantiles of the draws are the inversion band of l_k carried through equation (13.5).

Building blocks. The draws and their weights (Section 13.2.2); the binding loads (13.4) to (13.7) at $\hat{\mathbf{z}}$ and in each draw; the gradients of the same equations; the covariance $\boldsymbol{\Sigma}$ of Section 13.2.1.

Table 13.2: The binding loads of the site's four elements at the design day, in site IT load: the design sheet's, the MAP value and the delta-form standard deviation, the median and 90 % band of the 20 000 draws, the exact posterior's median and the stated truth.

Element	Sheet [kW]	$Q_e^{\dagger}(\hat{\mathbf{z}})$ [kW]	Delta sd [kW]	Median [kW]	5 % point [kW]	95 % point [kW]	Exact median [kW]	Truth [kW]
Chillers	1 228.0	1 134.06	22.36	1 135.67	1 102.58	1 178.75	1 135.14	1 106.00
Pipe	1 001.9	1 126.81	5.29	1 126.82	1 118.12	1 135.68	1 126.86	1 127.31
Feed	1 202.6	1 188.23	3.15	1 188.31	1 184.05	1 194.90	1 188.32	1 184.66
Pump	1 155.2	1 272.00	10.66	1 272.08	1 254.48	1 289.70	1 272.31	1 250.37

The four binding loads. Table 13.2 gives, for each element, the design sheet's value, the binding load at $\hat{\mathbf{z}}$ with its delta-form standard deviation, the median and the ends of the 90 % band by the draws, the exact posterior's median, and the stated truth of the record, a check marker for Section 13.4.3. The four posteriors are drawn on one axis in Figure 13.2: the horizontal axis is the site IT load, the vertical axis the density of the delta form of each element; the bars above the curves are the 90 % bands of the draws, with the plant's joint band at the top; the dash-dotted vertical lines at the bottom are the design sheet's four values, and the black dots on the bars the stated truth.

Three findings stand out, and the first is the one the design sheet did not expect. The pipe, which the sheet put first by 153 kW at 1 001.9 kW, binds at a median of 1 126.82 kW with a band from 1 118.12 to 1 135.68 kW: the record's twelve months of pressure drop moved its flow coefficient above the handbook's, and with it the pipe's binding load up by 125 kW, into the chillers' band. The chillers, which the sheet put last at 1 228.0 kW, bind at a median of 1 135.67 kW: their as-built ratios lie below nameplate, and they move down by 92 kW. The two now sit within 9 kW of each other at their medians, the pipe's band entirely inside the chillers'. The feed and the pump follow at 1 188.31 and 1 272.08 kW; the pump, which the sheet put second, is last.

The second finding is the spreads. The pipe and the feed are narrow, 5.29 and 3.15 kW in the delta form, because the monthly pressure drops and powers read their entries closely; the chillers' standard deviation of 22.36 kW is four times the pipe's, because its entries are known less closely: the three as-built ratios to about 0.017 each and the economizer's to 0.35. The third is the shape of the chillers' band. Its delta form is $1\,134.06 \pm 1.645 \times 22.36$, the band $[1\,097.28,\ 1\,170.84]$ kW; the draws put it at $[1\,102.58,\ 1\,178.75]$ kW, with the median 1.61 kW above the MAP value. The economizer's duty $50\,\mathrm{e}^{g_e}$ is an exponential of a normal entry of spread 0.35, skewed to the right, and the symmetric normal of the delta form places both ends too low. The other three delta bands agree with the draws more closely: the pipe's, $1\,126.81 \pm 1.645 \times 5.29 = [1\,118.11,\ 1\,135.52]$ kW, to within 0.16 kW at each end; the pump's, $[1\,254.45,\ 1\,289.54]$ kW, to within 0.16 kW; and the feed's, $[1\,183.05,\ 1\,193.41]$ kW, to within 1.49 kW. The exact posterior moves no median by more than 0.53 kW, the chillers', from 1 135.67 to 1 135.14 kW, and no end of a band by more than 0.62 kW: its 90 % bands are $[1\,102.07,\ 1\,178.13]$ kW for the chillers, $[1\,118.21,\ 1\,135.79]$ kW for the pipe, $[1\,184.06,\ 1\,194.91]$ kW for the feed and $[1\,254.93,\ 1\,290.16]$ kW for the pump.

The plant's binding load. The plant's binding load is the smallest of the four in each draw, and the draws give the reported value,

$$\text{median } Q^{\dagger} = 1\,123.88\,\text{kW}, \qquad Q^{\dagger} \in [1\,102.58,\ 1\,133.97]\,\text{kW with } 90\,\%\text{ probability}, \tag{13.10}$$

in site IT load. The exact posterior gives the median 1 123.82 kW and the band from 1 102.07 to 1 133.99 kW, a gap of at most 0.51 kW. Table 13.3 sets beside them the routes on one element alone: each row is a route, the columns the median and the ends of the band. The element that

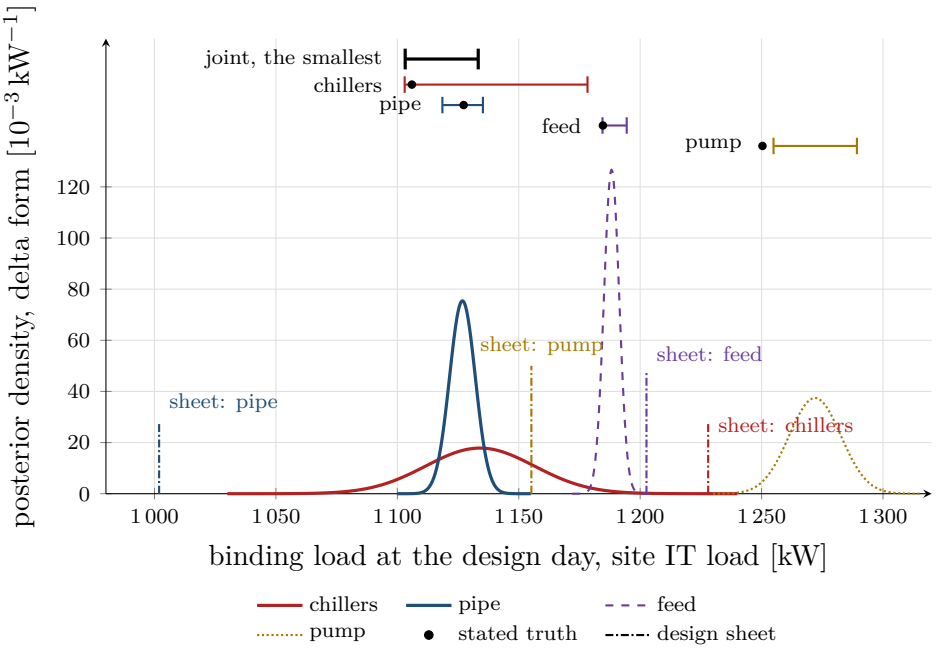


Figure 13.2: The binding loads of the site's four elements at the design day. Curves: the delta form of each element, $\mathcal{N}(Q_e^{\dagger}(\hat{\mathbf{z}}),\ \mathbf{g}_e^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_e)$, from the MAP values and standard deviations of Table 13.2. Bars: the 90 % bands of the 20 000 draws; the top bar is the plant's joint binding load, the smallest of the four in each draw, [1 102.58, 1 133.97] kW. Dots: the stated truth of the record, a check marker (Section 13.4.3). Dash-dotted lines at the bottom: the design sheet's values from nameplate and handbook ratings.

binds at the MAP state is the pipe, and its band alone, [1 118.12, 1 135.68] kW, misplaces the plant's at both ends. At the lower end the joint band reaches down to 1 102.58 kW, the chillers' own 5 % point to the hundredth of a kilowatt: below the pipe's lowest values only the chillers bind, so the lower tail of the minimum is the chillers' lower tail. At the upper end the joint band stops at 1 133.97 kW, short of the pipe's 1 135.68 kW, because in the draws where the pipe is high the chillers bind first and cut the tail. The gap between the pipe's band and the joint band is the chillers' share, at both ends.

13.3.2 The element that binds first

Planner's question. Which element binds first as the load grows: the chiller, the pump, the pipe, or the power feed? The question asks for the probability of each element being the one whose limit is met first.

General form. A prediction under a threshold is a tail of a derived quantity under the posterior, equation (2.12),

$$\Pr(f(\mathbf{z}) \geq f^* \mid \mathbf{y}) = \int \mathbf{1}\big[f(\mathbf{z}) \geq f^*\big]\, p(\mathbf{z} \mid \mathbf{y})\, \mathrm{d}\mathbf{z},$$

asked here over a family of events, one per element.

Table 13.3: The plant's binding load at the design day by the joint draws, beside the routes on the pipe alone and on the chillers alone, each with the exact posterior, in site IT load.

Route	Median [kW]	5 % point [kW]	95 % point [kW]
Joint draws, the smallest of the four (reported)	1 123.88	1 102.58	1 133.97
Joint, exact posterior	1 123.82	1 102.07	1 133.99
Pipe alone, draws (its inversion band)	1 126.82	1 118.12	1 135.68
Pipe alone, exact posterior	1 126.86	1 118.21	1 135.79
Pipe alone, delta form	1 126.81	1 118.11	1 135.52
Chillers alone, draws	1 135.67	1 102.58	1 178.75
Chillers alone, exact posterior	1 135.14	1 102.07	1 178.13
Chillers alone, delta form	1 134.06	1 097.28	1 170.84

Table 13.4: The probability that each element of the site binds first at the design day, equation (13.11), from 20 000 draws of the Laplace posterior and from the same draws reweighted to the exact posterior.

Element	$Q_e^\dagger(\hat{\mathbf{z}})$ [kW]	P_j, draws [–]	P_j, exact posterior [–]
Pipe	1 126.81	0.6505	0.6411
Chillers	1 134.06	0.3494	0.3589
Feed	1 188.23	< 0.0001	< 0.0001
Pump	1 272.00	< 0.0001	< 0.0001

Specialized form. For element j the question is equation (2.12) with $f(\mathbf{z}) = \min_{e \neq j} Q_e^\dagger(\mathbf{z}) - Q_j^\dagger(\mathbf{z})$ and $f^* = 0$, the event that j's binding load is the smallest, as in equation (5.22),

$$P_j = \Pr(e^\dagger(\mathbf{z}) = j \mid \mathbf{y}) \approx \hat{P}_j = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}[e^\dagger(\mathbf{z}_i) = j], \qquad \mathrm{se}(\hat{P}_j) = \sqrt{\hat{P}_j\,(1 - \hat{P}_j)/N}, \quad (13.11)$$

with se the Monte Carlo standard error; the exact posterior sums the weights w_i of the same draws. The event has no gradient, so the delta form cannot read it directly. It can read the one pair that matters: if only the pipe and the chillers ever bind first, the pipe binds first when the gap $D = Q_{\text{pipe}}^\dagger - Q_{\text{ch}}^\dagger$ of equation (13.3) is negative, and the delta form puts that probability at

$$\Pr(D < 0 \mid \mathbf{y}) \approx \Phi\left(\frac{-D(\hat{\mathbf{z}})}{\sqrt{\mathbf{g}_D^\mathsf{T} \boldsymbol{\Sigma} \mathbf{g}_D}} \right), \quad (13.12)$$

the single-pair route, which assumes the gap is normal.

Building blocks. The draws and their weights (Section 13.2.2); the binding element of each draw; the gap's MAP value and gradient, equations (13.3) and (13.1); the normal distribution function Φ.

The binding element. Table 13.4 gives the probabilities: each row is an element, the columns its binding load at $\hat{\mathbf{z}}$ and the probability that it binds first by the draws and by the exact posterior.

The pipe binds first with probability

$$\Pr(e^\dagger = \text{pipe} \mid \mathbf{y}) = 0.6505, \quad (13.13)$$

with a Monte Carlo standard error of $\sqrt{0.6505 \times 0.3495/20\,000} = 0.0034$, and the chillers with probability 0.3494. The exact posterior gives 0.6411 and 0.3589, a gap of -0.0094 for the pipe. The feed and the pump bind first in fewer than one draw in ten thousand: the question is a contest between two elements. The single-pair route (13.12) gives $\Phi(7.25/22.98) = \Phi(0.32) = 0.62$, with $D(\hat{\mathbf{z}}) = 1\,126.81 - 1\,134.06 = -7.25$ kW and the delta-form standard deviation of the gap, 22.98 kW. It sits below the draws for the reason the chillers' band does: the gap carries the chillers' right skew as a left skew, so its median by the draws, -8.84 kW, lies further below zero than its MAP value, and a symmetric normal places less of it below zero.

The design sheet's answer was the pipe, and the analysis agrees on the element and disagrees on everything that would justify acting on it. The sheet's pipe at 1 001.9 kW is the handbook's pipe; the record moved it to 1 126.8 kW, beside the chillers, and the pipe binds first in two draws of three, not with the certainty a 153 kW lead suggests.

13.3.3 The order of the four

Planner's question. What is the probability that the order flips? The question asks how often the four elements bind in an order other than the order at the MAP state, and which orders the posterior gives.

General form. A prediction under a threshold over a family of events, equation (2.12),

$$\Pr(f(\mathbf{z}) \geq f^* \mid \mathbf{y}) = \int \mathbf{1}\big[f(\mathbf{z}) \geq f^*\big]\, p(\mathbf{z} \mid \mathbf{y})\, \mathrm{d}\mathbf{z},$$

one event per order of the four elements.

Specialized form. The question is equation (2.12) with the indicator of the event $\pi(\mathbf{z}) = \sigma$ for each order σ of the four elements, by equation (13.2), and with the indicator of $\pi(\mathbf{z}) \neq \pi(\hat{\mathbf{z}})$ for the flip,

$$P_\sigma = \Pr\big(\pi(\mathbf{z}) = \sigma \mid \mathbf{y}\big), \qquad P_{\text{flip}} = \Pr\big(\pi(\mathbf{z}) \neq \pi(\hat{\mathbf{z}}) \mid \mathbf{y}\big) = 1 - P_{\pi(\hat{\mathbf{z}})}, \tag{13.14}$$

each read as the share of the draws in which its event holds, and by the exact posterior as the summed weights of those draws. A flip need not change the binding element: an exchange lower in the order, such as the feed passing the chillers, is a flip too.

Building blocks. The ranking of each draw (Section 13.2.2); the ranking at the MAP state; the weights.

The orders. At the MAP state the order is pipe, chillers, feed, pump, by the binding loads of Table 13.4. Table 13.5 lists every order the draws produce, with its probability, and the same probabilities are drawn in Figure 13.3, one bar per order, the MAP order marked.

The order flips with probability

$$P_{\text{flip}} = 0.3658, \tag{13.15}$$

with a Monte Carlo standard error of 0.0034; the exact posterior gives 0.3744. Four orders occur, and the bars tell their story. The pump is last in every order but the rarest. Almost the whole of the flip is the exchange of the first two: the chillers bind first in 0.3494 of the draws, the probability of the second bar, and in each of those the rest of the order is unchanged. The remaining $0.3658 - 0.3494 = 0.0164$ is the feed passing the chillers behind the pipe, in the draws where the economizer's guess is large: a large economizer raises the chillers' binding load by its duty and the feed's only by the chiller power that duty saves, so the chillers move past the feed. One question of the order is settled, the pump last, and one is open, the first.

Table 13.5: The orders in which the site's four elements bind at the design day, smallest binding load first, with their probabilities over the 20 000 draws. The first row is the order at the MAP state.

Order [–]	Probability [–]
pipe, chillers, feed, pump (the MAP order)	0.6342
chillers, pipe, feed, pump	0.3494
pipe, feed, chillers, pump	0.0163
pipe, feed, pump, chillers	0.0001
any order other than the MAP order, draws (reported)	0.3658
any order other than the MAP order, exact posterior	0.3744

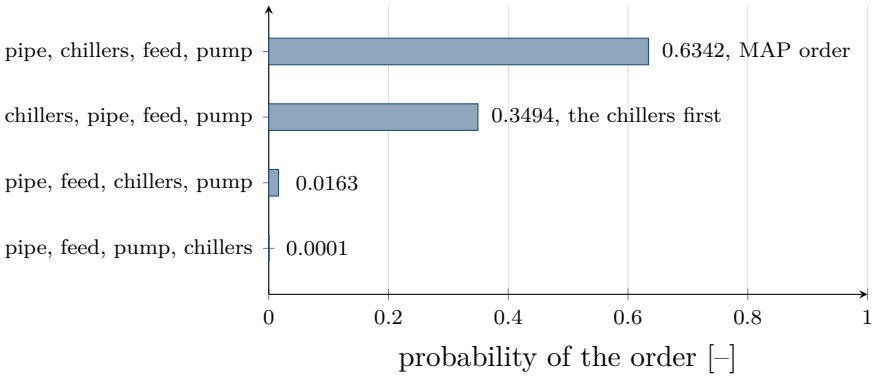


Figure 13.3: The probability of each order of the site's four elements at the design day, from the 20 000 draws (Table 13.5). The top bar is the order at the MAP state; every other bar is a flip, and together they make 0.3658.

Table 13.6: The variance budget of the gap between the pipe's and the chillers' binding loads at the design day, equation (5.25), at the MAP state, in shares of $\mathbf{g}_D^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_D = 22.98^2\,\mathrm{kW}^2$.

Entry [unit]	Enters the binding load of [–]	Share [–]
g_e, the economizer's log-ratio [–]	chillers (and feed)	0.580
κ_{C3} [–]	chillers	0.136
κ_{C2} [–]	chillers	0.135
κ_{C1} [–]	chillers	0.095
l_k, the pipe's log flow coefficient [–]	pipe (and pump, feed)	0.053
d, c, l_e [K^{-1}, –, –]	none of the two	0.000

13.3.4 What decides between the pipe and the chillers

Planner's question. What decides between the pipe and the chillers? The question asks which entries of the state, and so which parts of the evidence, carry the band of the gap between the two elements that contest the first place.

General form. The variance of a derived quantity under the posterior, equation (2.10),

$$\mathrm{Var}(f(\mathbf{z}) \mid \mathbf{y}) = \int \left(f(\mathbf{z}) - \mathbb{E}[f(\mathbf{z}) \mid \mathbf{y}]\right)^2 p(\mathbf{z} \mid \mathbf{y})\,\mathrm{d}\mathbf{z},$$

split into one share per entry.

Specialized form. The question is equation (2.10) with $f(\mathbf{z}) = D(\mathbf{z}) = Q^{\dagger}_{\mathrm{pipe}}(\mathbf{z}) - Q^{\dagger}_{\mathrm{ch}}(\mathbf{z})$, the gap of equation (13.3), on the Laplace normal, where its variance is $\mathbf{g}_D^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_D$ with $\mathbf{g}_D = \mathbf{g}_{\mathrm{pipe}} - \mathbf{g}_{\mathrm{ch}}$. It splits into one share per entry by the covariance split (5.25), $s_i = g_{D,i}\,(\boldsymbol{\Sigma}\mathbf{g}_D)_i / \mathbf{g}_D^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{g}_D$. By the pattern (13.8) the gradient has entries in the three log-ratios, in g_e and in l_k, and none in d, c or l_e, whose shares are zero. The entry g_e has no covariance with any other, so its share is its diagonal term alone, $(Q_e(\hat{\mathbf{z}}) \times 0.35)^2$ over the gap's variance, with $Q_e(\hat{\mathbf{z}}) = 50\,\mathrm{e}^0 = 50\,\mathrm{kW}$ by equation (13.4).

Building blocks. The gradients (13.4) and (13.5) at $\hat{\mathbf{z}}$; the covariance $\boldsymbol{\Sigma}$; the split (5.25); the draws for the gap's quantiles.

The gap and its budget. The gap is $-7.25\,\mathrm{kW}$ at the MAP state, with a delta-form standard deviation of $22.98\,\mathrm{kW}$; by the draws its median is $-8.84\,\mathrm{kW}$ and its 90 % band $[-53.00,\ 25.29]\,\mathrm{kW}$, and its share below zero is 0.6505, the pipe's probability of binding first. The standard deviation is nearly the two elements' combined, $22.36^2 + 5.29^2 = 527.95\,\mathrm{kW}^2$ against $22.98^2 = 528.08\,\mathrm{kW}^2$: the covariances between the chillers' entries and the pipe's are near zero. Table 13.6 splits the variance: each row is an entry of the state, the columns the element whose binding load it enters and its share.

The economizer's handbook guess carries 0.580 of the variance, the diagonal term $(50 \times 0.35)^2 = 306.25\,\mathrm{kW}^2$ over $528.08\,\mathrm{kW}^2$. The three chillers' as-built ratios carry 0.366 together, and the pipe, the element the sheet would upgrade, 0.053. This is the surprise of the chapter: the order of the two elements that contest the first place is decided mostly by a guess about a third element that is not built. Neither the pipe nor the chillers are in doubt to that degree; the twelve months of pressure drop fix the pipe to $5.29\,\mathrm{kW}$, and the capacity tests fix each existing unit's log-ratio to under 0.02. What is in doubt is the duty the drawn economizer will add to the chillers, $50\,\mathrm{kW}$ at the handbook value with a spread of 35 %, and that alone is enough to put the chillers on either side of the pipe. The entries that carry a plant's band, and how much of it a measurement of each would remove, are the subject of Chapter 17.

13.4 The checks (step 5)

The answers of Section 13.3 rest on the drawn economizer, on the Laplace normal of the site's fit and on a record generated from a stated truth; each is tested here.

13.4.1 The plant as built today

The economizer is drawn but not built, so the plant as it stands has the chillers' binding load without the duty $50\,\mathrm{e}^{g_e}$. On the same draws, with the economizer's term removed from equation (13.4) and the other elements unchanged, the chillers bind at a median of 1 084.16 kW, with a 90 % band of [1 061.54, 1 107.32] kW, and they bind first with probability 0.9970; the pipe binds first in the remaining 0.0030, and the feed and the pump in fewer than one draw in ten thousand. Without the economizer the question has a clear answer, the chillers, and the pipe becomes a contender only through the duty the drawing adds. The design sheet's recommendation, read on the plant as built, would upgrade the element that binds first in three draws of a thousand.

13.4.2 The draws against the exact posterior

The reported answers average the 20 000 draws with equal weight; the exact posterior reweights the same draws, with an effective sample size of 19 913. The medians of the four binding loads move by −0.53, +0.04, +0.01 and +0.23 kW, exact minus reported, for the chillers, the pipe, the feed and the pump. The probabilities move more than the medians would suggest: the pipe's probability of binding first by −0.0094 and the flip by +0.0086, each between two and three Monte Carlo standard errors of 0.0034. The two estimates use the same states, so their difference is not sampling noise between independent sets: it is the departure of the posterior of the state from the Laplace normal, and this one has a visible cause. The exact posterior moves the chillers' median down by about half a kilowatt and the pipe's not at all, so the gap moves toward the chillers by about a fortieth of its 23 kW spread; at the center of the gap's distribution, where the probability of its sign is most sensitive, that is a shift of about a hundredth. Both readings put the pipe first in about two draws of three, and neither changes the report.

13.4.3 The answers against the stated truth

The operating record is generated from a stated truth, and the last check compares the answers with it. Table 13.7 collects the checks: each row is a quantity, the columns its reported value, the value beside it, the truth, and the reading. At the truth the chillers bind at 1 106.00 kW, the pipe at 1 127.31 kW, the feed at 1 184.66 kW and the pump at 1 250.37 kW, so the true order is chillers, pipe, feed, pump: not the MAP order, but the second most likely order of Table 13.5, to which the draws give 0.3494. The chillers' truth lies inside their band near its lower end: the true economizer is smaller than the handbook's, $g_e = -0.2231$ against the prior's 0, and the true units are a little weaker than their MAP ratios (Table 9.2). The pipe's truth lies inside its band, 0.49 kW above the median, and the feed's inside its band, 0.61 kW above its lower end. The pump's truth lies outside its band, 4.11 kW below the 5 % point: the true pump efficiency, $l_e = -0.3285$, lies $(-0.2891 + 0.3285)/0.0190 = 2.07$ posterior standard deviations below the MAP value, an excursion rare for one entry but not for one of eight. It does not touch the order, since the pump binds last by more than 60 kW either way.

The years. Mapped through the load path of equation (6.1) with today's site IT load of 660 kW and growth of 15 % a year, each binding load becomes a crossing year by equation (6.3), draw by draw: the pipe in year 3.83, band [3.77, 3.88]; the chillers in year 3.88, band [3.67, 4.15]; the feed in year 4.21, band [4.18, 4.25]; and the pump in year 4.69, band [4.60, 4.79], medians

Table 13.7: The checks of the binding-constraint answers. The reported values are from the 20 000 draws; beside them, the exact posterior or the route named; the truth is the stated truth of the record.

Quantity	Reported [kW or –]	Beside [kW or –]	Truth [kW or –]	Reading
$Q^{\dagger}_{\mathrm{ch}}$, median	1 135.67	exact 1 135.14	1 106.00	inside, near the lower end
$Q^{\dagger}_{\mathrm{pipe}}$, median	1 126.82	exact 1 126.86	1 127.31	inside
$Q^{\dagger}_{\mathrm{feed}}$, median	1 188.31	exact 1 188.32	1 184.66	inside, near the lower end
$Q^{\dagger}_{\mathrm{pump}}$, median	1 272.08	exact 1 272.31	1 250.37	below the band; l_e off by 2.07 sd
$Q^{\dagger}$, the plant, median	1 123.88	exact 1 123.82	1 106.00	inside, near the lower end
Pr(pipe first)	0.6505	exact 0.6411	–	truth: the chillers first
Pr(order flips)	0.3658	exact 0.3744	–	true order has 0.3494
Pipe first, single-pair delta form	–	0.62	–	below the draws; skew
The plant as built, without the economizer				
$Q^{\dagger}_{\mathrm{ch}}$, median	1 084.16	–	–	band [1 061.54, 1 107.32]
Pr(chillers first)	0.9970	–	–	the pipe 0.0030
Crossing year at 15 % *a year from* 660 kW*, median and* 90 % *band*				
Pipe [yr]	3.83, band [3.77, 3.88]			
Chillers [yr]	3.88, band [3.67, 4.15]			
Feed [yr]	4.21, band [4.18, 4.25]			
Pump [yr]	4.69, band [4.60, 4.79]			
Plant, the smallest [yr]	3.81, band [3.67, 3.87]			

with 90 % bands. The crossing year is increasing in the binding load, so the plant's year has the quantiles of the joint binding load (13.10), mapped: a median of $\ln(1\,123.88/660)/\ln 1.15 = 3.81$, with the band [3.67, 3.87]. The two contenders cross within three weeks of each other at their medians, and the pipe's whole band lies inside the chillers'; which of them is crossed first is the question of Section 13.3.2, not a matter of the calendar.

13.5 The report (step 6)

The design sheet named the right element for the wrong reason. On the design day, with every unit up and the drawn economizer, the pipe binds first with probability 0.65 and the chillers with probability 0.35, at a plant binding load of 1 124 kW of site IT load, band 1 103 to 1 134 kW, reached in year 3.8 of the load path. The order of the two is decided mostly by the economizer's handbook conductance, and an upgrade committed on the sheet's ranking would rest on that guess.

13.5.1 The answers and their bands

Table 13.8 gives the answers in the order of Table 13.1. Each row is a question; the second column gives the reported answer with the exact posterior beside it, and the third its band or the routes beside it.

13.5.2 The design-review entry

The design review's entry replaces the sheet's ranking with a probability. On the design day the site's binding constraint is the shared pipe with probability 0.65 by the draws, 0.64 by the exact posterior, and the chillers with the drawn economizer otherwise; the feed and the pump bind later in every draw. The plant's binding load is 1 123.9 kW of site IT load at the median,

Table 13.8: The report for the site's four elements at the design day, with all units up and the drawn economizer, in site IT load. Reported values by 20 000 draws of the posterior of the state, joint over the four elements; the exact posterior beside each.

Question	Answer	Band or routes beside
Which element binds first as the load grows: the chiller, the pump, the pipe, or the power feed?	the pipe, probability 0.6505 (exact: 0.6411); the chillers 0.3494 (exact: 0.3589); the feed and the pump below 0.0001	Monte Carlo standard error 0.0034; single-pair delta form 0.62; without the economizer the chillers, 0.9970
At what load does each bind, and how sure?	plant, the smallest: median 1 123.9 kW (exact: 1 123.8); pipe 1 126.8, chillers 1 135.7, feed 1 188.3, pump 1 272.1 kW (medians)	plant 1 102.6 to 1 134.0 kW (exact: 1 102.1 to 1 134.0); pipe 1 118.1 to 1 135.7; chillers 1 102.6 to 1 178.8; feed 1 184.1 to 1 194.9; pump 1 254.5 to 1 289.7 kW; central 90 %
What is the probability that the order flips?	0.3658 (exact: 0.3744)	chillers, pipe, feed, pump 0.3494; pipe, feed, chillers, pump 0.0163
What decides between the pipe and the chillers?	the economizer's handbook guess, 0.580 of the gap's variance	the three chillers' ratios 0.366; the pipe 0.053; gap median −8.8 kW, band −53.0 to 25.3 kW

band 1 102.6 to 1 134.0 kW, crossed in year 3.81 of the load path, band 3.67 to 3.87. The entry records three further facts. The sheet's pipe at 1 001.9 kW was the handbook's; the record puts it at 1 126.8 kW, and an upgrade of the pipe alone would leave the chillers binding at a median of 1 135.7 kW, 9 kW above where the pipe binds now; what an upgrade is worth is Chapter 14's question. Without the economizer the chillers bind first with probability 0.997. And 58 % of the variance of the gap that decides the order is the economizer's handbook conductance, which no reading of the existing plant sees. The entry's decision follows: no upgrade is committed on the sheet's ranking, and the economizer's conductance is measured or specified before the order is relied on.

Common mistake

Ranking the bottlenecks by nameplate and handbook values. Each binding load moves by what the record knows of its own element, and the order of two close elements is decided by whichever entry carries the band of their gap, which may belong to neither.

13.6 The next measurement (step 7)

The report leaves the order of the two leading elements on a coin weighted about two to one, and the weight is set by one prior. Step 7 asks which test or survey would narrow the quantity that decides the build; this chapter names the entry and points forward, and computes nothing new.

13.6.1 The entry that decides the order

The gap's budget names the entry: the economizer's log-ratio g_e, with 0.580 of the variance. A measurement that fixed it would leave the gap with the remaining 0.420, carried by the chillers' as-built ratios and the pipe. No test of the existing plant can see the economizer, since it is not built: its conductance can be fixed only by the manufacturer's rating test of the selected unit, by a pilot, or by a specification that makes the handbook value a contractual minimum. The chillers' share, 0.366, is open to a capacity test of each unit at the design-day wet bulb. The pipe's share, 0.053, is already small: another year of pressure-drop readings would buy little.

Chapter 17 splits a plant's band by its entries and states what each costs the decision, and Chapter 18 ranks the tests and surveys a planner can commission by the width each takes off the quantity that decides the build; the economizer's conductance heads this chapter's list.

13.6.2 What the analysis showed

The design sheet put the pipe first by 153 kW and recommended an upgrade. The analysis puts the pipe first too, with probability 0.65, but 9 kW ahead of the chillers at the medians, not 153 kW, and with the order reversed in a third of the posterior. Three findings carry the answer. First, the record moved the two elements toward each other from opposite sides: the pipe passes more flow than its handbook coefficient, and the chillers deliver less than nameplate, so the sheet's comfortable lead was an artifact of two handbook values. Second, the joint binding load differs from the binding element's: the plant's band reaches down to the chillers' lower tail and stops short of the pipe's upper tail, because the chillers bind wherever the pipe is high. Third, and beyond this site, the order of two close elements can be decided by an element that is neither. Here the drawn economizer's guessed conductance carries 58 % of the gap's variance; without it the question has a clear answer, the chillers, and with it the answer turns on a number no reading of the plant contains.

13.6.3 Limits of the model

The design day with all units up. The four binding loads are read at the design day's wet bulb with every chiller running. With a unit out the chillers' binding load falls by that unit's capacity, some 350 kW or more, and the chillers bind long before the pipe; with the shared pipe itself out of service the plant carries nothing. Chapter 15 brings the outages and the halls into the same reading.

One efficiency per pump, one coefficient per pipe. The pump has one efficiency at every flow, and the pipe one flow coefficient with a drop that rises with the square of the flow. Near its motor rating a real pump runs away from its best point, and a pipe's drop includes fittings whose loss coefficients change with the flow; a pump curve and a measured system curve would replace both entries.

Fixed supply, rise and power factor. The supply temperature, the loop's rise and the feed's power factor are pinned. A plant that raises its supply temperature at high load moves heat into a smaller flow and relieves the pipe and the pump; a falling power factor would lower the feed's limit.

The economizer at its handbook value. The economizer enters every answer through a prior alone. Its truth in this record is smaller than the handbook's, which is why the true order puts the chillers first; a different guess would move the probabilities of Table 13.4 in either direction.

The site's elements only. The binding loads are those of the central plant's four elements. A rack inlet in a hall, the limit of Chapter 5, can bind before any of them, and the hall units' capacities are not in this chapter's state.

The constructed record. The operating record and the capacity tests are generated from a stated truth with the stated reading errors, not taken from a site, and the comparison with that truth in Section 13.4.3 is a check of the method on its own model, not of the model against a real plant.

The next chapter. Chapter 14 asks what raising each element's limit is worth: which upgrade moves the plant's binding load most, by the importance of each rating.

13.7 What to retain

Definition 13.1 (Binding constraint and its ranking)**.** The binding load $Q_e^\dagger$ of each element e of a plant is the scenario solved for the load at which its limit is met, $h_e(\mathbf{z}, Q_e^\dagger) = h_e^{\max}$, with the implicit-function gradient $\mathbf{g}_e = -\nabla_{\mathbf{z}} h_e / (\partial h_e / \partial Q^*)$ in kilowatts of load. The binding constraint is the smallest binding load, the binding element the one that attains it, and the ranking the order of all of them. The binding load is reported jointly, from draws of the posterior of the state pushed through every element; the binding element and the ranking have no gradient and are reported as probabilities over the same draws, with the exact posterior beside them. The order of two elements is the sign of their gap, whose variance budget names the entries that decide it.

Four conclusions carry beyond this site. A ranking of bottlenecks is a probability distribution over orders, not a list, and the probability that it flips is a number the review can read. Elements ranked on handbook values can trade places once the record is read: each binding load moves by what the record knows of its own element, and two that look far apart may be close. The plant's binding load is the joint minimum, whose tails belong to whichever element binds there, not to the element that binds at the MAP state. And the order of two close elements is decided by the entry that carries the band of their gap, which can be an unmeasured guess about a third element; that entry, not either contender, is what to measure before an upgrade is chosen.

Table 13.9 lists the terms of the chapter with their expressions, units and the subsection that defines each.

Table 13.9: The terms of binding constraints and where each is defined. Units for the site at the design day.

Term	Expression	Units	Meaning	Defined in
Element's binding load	$h_e(\mathbf{z}, Q_e^\dagger) = h_e^{\max}$; $\mathbf{g}_e = -\nabla_{\mathbf{z}} h_e / (\partial h_e / \partial Q^*)$	kW	limit of one element	13.1.1
Binding element	$e^\dagger = \arg\min_e Q_e^\dagger$	–	the element that binds first	13.1.1
Ranking	$\pi = \operatorname{argsort}_e Q_e^\dagger$	–	the order of the elements	13.1.1
Pairwise gap	$D_{ab} = Q_a^\dagger - Q_b^\dagger$	kW	its sign is the order	13.1.1
Chillers	$\sum_i 400\,\mathrm{e}^{\kappa_i} + 50\,\mathrm{e}^{g_e} - Q_o$	kW	heat the plant can carry	13.1.3
Pipe	$\rho c_p \Delta T_w \sqrt{\Delta P_{\max}}\,\mathrm{e}^{l_k} - Q_o$	kW	flow at the allowable drop	13.1.3
Pump, feed	roots of $h(V) = P_m$, $F(Q^*) = 0$	kW	motor and feed limits	13.1.3
Gradient pattern	eq. (13.8)	–	the entries each element reads	13.1.3
Questions and answers				
Joint binding load	quantiles of $\min_e Q_e^\dagger(\mathbf{z}_i)$	kW	the reported plant value	13.3.1
Binding-element probability	$\Pr(e^\dagger = j \mid \mathbf{y})$	–	which element binds first	13.3.2
Single-pair route	$\Phi(-D(\hat{\mathbf{z}}) / \sqrt{\mathbf{g}_D^\mathsf{T} \boldsymbol{\Sigma} \mathbf{g}_D})$	–	delta form on the gap	13.3.2
Order flip	$\Pr(\pi(\mathbf{z}) \neq \pi(\hat{\mathbf{z}}) \mid \mathbf{y})$	–	the order is not the MAP order	13.3.3
Gap budget	$s_i = g_{D,i} (\boldsymbol{\Sigma} \mathbf{g}_D)_i / \mathbf{g}_D^\mathsf{T} \boldsymbol{\Sigma} \mathbf{g}_D$	–	what decides the order	13.3.4

13.8 Exercises

Exercise 13.1. *Formulation.* The planner asks how likely it is that the feed binds before the chillers. Write the question as a probability over the state and state its answer from this chapter's draws. ($\Pr(Q^{\dagger}_{\text{feed}}(\mathbf{z}) < Q^{\dagger}_{\text{ch}}(\mathbf{z}) \mid \mathbf{y})$, equation (2.12) with the gap $D = Q^{\dagger}_{\text{feed}} - Q^{\dagger}_{\text{ch}}$ of equation (13.3) and $f^* = 0$. It is the summed probability of the orders in Table 13.5 in which the feed comes before the chillers: pipe, feed, chillers, pump at 0.0163 and pipe, feed, pump, chillers at 0.0001, together 0.0164.)

Exercise 13.2. *Derivation.* Derive the pipe's binding load and its gradient from the pressure-drop relation of equation (9.3), and check its delta-form standard deviation. (The drop $(V/k_p)^2$ meets $\Delta P_{\max}$ at $V = k_p\sqrt{\Delta P_{\max}}$; the loop's heat at that flow is $\rho c_p\,\Delta T_w\,V$, and the site IT load is that less Q_o, so $Q^{\dagger}_{\text{pipe}} = \rho c_p\,\Delta T_w\sqrt{\Delta P_{\max}}\,\mathrm{e}^{l_k} - Q_o$. Its only gradient entry is $\partial Q^{\dagger}_{\text{pipe}}/\partial l_k = Q^{\dagger}_{\text{pipe}} + Q_o = 1\,126.81 + 22 = 1\,148.81$ kW, and with $\mathrm{sd}(l_k) = 0.0046$ the standard deviation is $1\,148.81 \times 0.0046 = 5.28$ kW, against the printed 5.29 kW from the unrounded spread.)

Exercise 13.3. *Derivation.* Show that the pipe's binding load scales with the flow coefficient, and use the design sheet and the fit to check the MAP value. (By equation (13.5), $Q^{\dagger}_{\text{pipe}} + Q_o$ is proportional to e^{l_k}. The sheet takes the handbook's $l_k = \ln 2 = 0.6931$ and finds $1\,001.9 + 22 = 1\,023.9$ kW of loop heat; the fit's $l_k = 0.8083$ multiplies that by $\mathrm{e}^{0.1152} = 1.122$, giving about 1 148.9 kW of loop heat and 1 126.9 kW of site IT load, against the printed 1 126.81 kW from the unrounded entry.)

Exercise 13.4. *Application.* Show from the gradient and the covariance why the economizer's share of the gap's budget is 0.580. (The economizer's entry g_e has no row but its prior, so it is uncorrelated with every other entry and its share is its diagonal term alone. Its gradient entry in the gap is $-Q_e(\hat{\mathbf{z}}) = -50\,\mathrm{e}^0 = -50$ kW, by equation (13.4), and its variance 0.35^2, so the term is $(50 \times 0.35)^2 = 306.25\,\mathrm{kW}^2$; over the gap's variance $22.98^2 = 528.08\,\mathrm{kW}^2$ it is 0.580.)

Exercise 13.5. *Application.* The facilities team proposes to drop the economizer from the drawing to save capital. Say which element then binds first, at about what load, and how sure. (Without the economizer the chillers' binding load loses $50\,\mathrm{e}^{g_e}$, 50 kW at the MAP state, so that $1\,134.06 - 50 = 1\,084.06$ kW there, below the pipe's 1 126.81 kW by more than 40 kW, with the economizer's share of the chillers' band removed with it. By the draws the chillers bind at a median of 1 084.16 kW, band 1 061.54 to 1 107.32 kW, and they bind first with probability 0.9970.)

Chapter 14

Capacity Importance Measures

14.1 Which upgrade buys the most headroom

The site's design review has a budget for one upgrade, and four are priced. The chiller plant can take a fourth 400 kW chiller for R$ 1.60 million; the shared supply pipe can be replaced one size up, which raises its flow coefficient by a quarter, for R$ 0.90 million; the pump's motor can go from 12 to 18 kW for R$ 0.15 million; and the power feed from 1 600 to 2 000 kVA for R$ 0.80 million. Each buys some rating of one element, and the review wants to know which buys the most headroom over today's 660 kW of site IT load.

The design sheet ranks the upgrades by its own binding order. Worked from the nameplates and the handbook values at the design day, it finds that the shared pipe reaches its allowable pressure drop first, at 1 001.9 kW of site IT load; the pump reaches its motor's power at 1 155.2 kW, the feed its rating at 1 202.6 kW, and the chillers their capacity at 1 228.0 kW. The pipe binds first, so the sheet says to upgrade the pipe. Its logic is the logic of a bottleneck: the element that binds first is the one whose rating the headroom moves with, and it moves with no other. The planner's question is the drill question, verbatim,

If I could upgrade one element, which one buys the most headroom?

It is answered with the method of Chapter 4 on the site of Chapter 9, with the one fit of its operating record. Chapter 13 asks which element binds first and finds the answer uncertain; this chapter asks what that uncertainty is worth when a rating is bought. What is new is the quantity asked for: not a binding load, but how the headroom, the smallest of four binding loads, changes when one of them is moved by a stated step.

Jargon: importance measure, debottlenecking, rating, upgrade step, flow coefficient, power factor, binds first, derivative of a minimum, expected gain, complementary upgrades, gain per real

An *importance measure* ranks the elements of a plant by how much a quantity of the whole plant changes with each of them; in the capacity sense used here, it is the change of the headroom per unit of an element's rating. *Debottlenecking* is the practice of raising the rating of the element that limits a plant. The *rating* r_e of an element e is the number on its nameplate that sets its limit: the chillers' total nameplate in kW, the pipe's flow coefficient, the pump motor's power in kW and the feed's rating in kVA. An *upgrade step* Δ_e is a stated change of one rating with a stated price. The *flow coefficient* k_p of a pipe relates its flow to its pressure drop, $\Delta P = (V/k_p)^2$; a pipe one size up has a larger coefficient. The *power factor* of the feed is the ratio of the real power it delivers to its apparent power in kVA. An element *binds first* when its binding load is the smallest of the plant's. The *derivative of a minimum* of several smooth functions is, wherever one of them is strictly the smallest, the derivative of that one. The *expected gain* of an upgrade is the change of the headroom it buys, averaged over the posterior. Two upgrades are *complementary* when they buy more together than the sum of what each buys alone. The *gain per real* is the expected gain divided by the price, here in kW per million reais (R$ M).

Key idea

The headroom of a plant is the smallest of its elements' binding loads less today's load, and an importance measure is its derivative in one element's rating. That derivative is the element's own gradient where the element binds first and zero elsewhere, so over the posterior it is the probability that the element binds first times its gradient. It is local. A stated upgrade is a step, and the headroom gained by a step stops growing where the next element binds; the gain is read jointly from draws of the posterior with every element's binding load, before and after the step. The importance measure says where to look, and the joint gain says what to buy.

14.1.1 Importance read as the derivative of a minimum

The headroom of the site is set by its four elements together: the chillers, the shared pipe, the pump and the feed. Each element e has a binding load $Q_e^\dagger$, the site IT load at which its limit binds, in the sense of Section 5.1.1; it depends on the plant's state $\mathbf{z}$ and on the element's own rating r_e. The headroom is the smallest binding load less today's load,

$$H(\mathbf{z}, \mathbf{r}) = \min_e Q_e^\dagger(\mathbf{z}, \mathbf{r}) - Q_0, \qquad e_1(\mathbf{z}, \mathbf{r}) = \arg\min_e Q_e^\dagger(\mathbf{z}, \mathbf{r}), \tag{14.1}$$

with $\mathbf{r} = (r_{\mathrm{ch}}, r_{\mathrm{p}}, r_{\mathrm{m}}, r_{\mathrm{f}})$ the four ratings, $Q_0 = 660\,\mathrm{kW}$ today's site IT load, a stated input, and e_1 the element that binds first. Write $g_e = \partial Q_e^\dagger / \partial r_e$ for the gradient of element e's binding load in its own rating, in kW of site IT load per unit of the rating.

Three readings of an element's importance follow from equation (14.1). The first is the derivative of the headroom in the rating at one state. Wherever one element binds strictly first, the minimum moves with that element alone, so the derivative is that element's gradient, and the other elements' is zero. At the MAP state $\hat{\mathbf{z}}$ of the site's fit,

$$I_e^{\mathrm{MAP}} = \left.\frac{\partial H}{\partial r_e}\right|_{\hat{\mathbf{z}}} = g_e(\hat{\mathbf{z}})\,\mathbf{1}\big[e_1(\hat{\mathbf{z}}) = e\big]. \tag{14.2}$$

The second averages that derivative over the posterior. Ties between two binding loads have probability zero under a continuous posterior, so the derivative exists almost everywhere and

$$I_e = \mathbb{E}\Big[\frac{\partial H}{\partial r_e} \,\Big|\, \mathbf{y}\Big] = \int g_e(\mathbf{z})\,\mathbf{1}\big[e_1(\mathbf{z}) = e\big]\, p(\mathbf{z} \mid \mathbf{y})\,\mathrm{d}\mathbf{z} = \Pr(e_1 = e \mid \mathbf{y})\,\mathbb{E}\big[g_e \mid e_1 = e,\ \mathbf{y}\big], \tag{14.3}$$

END OF THE PREVIEW

Capacity Planning and Design by Derivation

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