Effect of workload balancing across data-center buildings on campus PUE and rack temperatures

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Abstract

The electricity a data center spends beyond its computing is summarized by its power usage effectiveness (PUE), the ratio of total electricity to the electricity reaching the computers, and operators often expect that evening out the computing load across the buildings of a campus lowers it. This report tests that expectation, together with the effect on rack temperatures, on a campus of three identical buildings of 30 racks each that share one chilled-water plant and carry 30, 20 and 10 kW of computing today. A balancing fraction moves each building's load linearly from today's split to equal loads, and at each of 51 settings the campus is solved as a steady thermal network of racks, aisles, rooms and buildings in which rack power rises with rack temperature, its conductances sized so that each link carries the heaviest rack's heat at a stated temperature difference and its boundary held at the 61 °F chilled water the plant supplies; a two-unit plant whose Gordon–Ng chiller law is anchored to the ASHRAE 90.1 minimum efficiency for its size prices the cooling. Campus PUE is 1.1638 at every setting: its largest change across the sweep is below 10^{-9}, against an invariance tolerance of 0.001, and total computing power, 60.39 kW, changes by less than 10^{-9} kW. The hottest rack on the campus falls by 7.7 K, from 312.0 K today to 304.4 K at equal loads, the best of the settings, and every rack stays between 296.7 and 312.0 K. PUE cannot move because every relation in the network is linear and the buildings are identical, so total computing power depends only on the total base load, which balancing preserves, and a plant at a fixed heat sink prices only its total duty. The invariance holds in 48 of 48 combinations of the plant's modeling choices, a sensitivity analysis over those choices. The study is a forward one: every input is stated and nothing is estimated from data, and every number comes from 51 steady solves.

1 Introduction

Data centers turn electricity into computation, and much of the electricity that does not reach the computers is spent on removing their heat. The industry's summary of that overhead is the power usage effectiveness, total facility electricity divided by the electricity delivered to the computing equipment, which Barroso, Hölzle and Ranganathan (2019) discuss as the facility-efficiency measure of warehouse-scale computers. A campus of several buildings on one cooling plant offers an operator a lever that needs no construction: moving computing work between buildings. Whether that lever changes the electricity bill, the thermal risk to the hardware, or both is not obvious, because the two are coupled. Computers draw more power when they run hot, the cooling plant's efficiency varies with its load, and a heavily loaded building runs hotter than a lightly loaded one.

Workload placement as a thermal-management tool has a long history. Sharma, Bash, Patel, Friedrich and Chase (2005) argued that where work runs inside a data center is itself a cooling decision and proposed redistributing it according to the thermal state of the room; Moore, Chase

and Ranganathan (2006) learned from sensor data a model that predicts the temperatures a placement will produce, so that placements can be judged before they are made. On the plant side, thermodynamic chiller models that tie a machine's power to its load and its water temperatures through a few physically meaningful loss coefficients descend from the work of Gordon, Ng and co-workers, including the experimental study of Chua, Ng and Gordon (1996), and the minimum efficiencies a new chiller must meet at the AHRI 550/590 rating conditions are set by ASHRAE Standard 90.1 (ASHRAE, 2019).

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses its data-center plugin to write a three-building campus as heat balances at racks, aisles, rooms and buildings joined by conductances, solves the steady state at each workload split, and prices the resulting cooling duty with its chiller-plant plugin. Section 3 states the framework and each of these constructs precisely.

This report asks whether evening out the computing load across the buildings of a campus changes the power bill, the hottest rack, or both. It answers with one thermal network solved across a sweep of workload splits and priced by a code-minimum plant, and it checks the answer against the plant's modeling choices. It is a forward study: every input is stated and nothing is estimated from data. It adds a mechanism that explains the result exactly rather than a trend fitted to it, and no claim of novelty is made. Section 2 describes the campus and the model, Section 3 the equations and the protocol, Section 4 the results, Section 5 their limits and Section 6 the answers.

2 System and data

The campus. Figure 1 draws the campus. Three buildings with identical hardware each hold 30 racks in 3 rooms, one row of 10 racks per room, with the same containment and the same cooling conductances. Their computing loads differ: building A carries 30 kW, building B 20 kW and building C 10 kW of base load, spread evenly over each building's racks. Each rack's air path runs from its row's cold aisle (the air the racks draw) through the rack to the hot aisle (the air they exhaust); part of the hot air recirculates into the cold aisle, the rest returns to the room's cooling, which passes the heat to the building's chilled-water coil.

The plant. One chilled-water plant serves all three buildings. It has 2 identical water-cooled positive-displacement units of 40 kW (11.4 tons of refrigeration), each the least efficient machine that meets both ASHRAE 90.1 minimums for its size, and it starts a second unit when the load passes 90 % of the running units' capacity. Its condenser water enters at 85 °F, the AHRI 550/590 full-load rating condition, at every setting: the campus has no weather. Chilled water enters the evaporators at 61 °F, the temperature at which the buildings reject their heat, and leaves at 51 °F.

The data. No measured data enter the study. It is a forward comparison: every input is a stated parameter, listed in Table 1, and every output is computed from them. Nothing is estimated, no noise is drawn, and a rerun reproduces the results file.

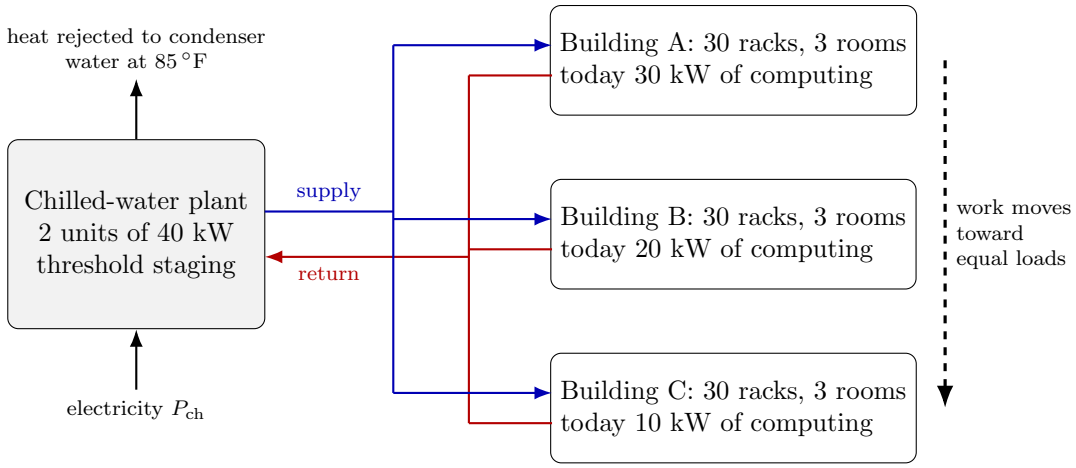


Figure 1: The campus. Three identical buildings of 30 racks share one chilled-water plant of 2 units of 40 kW (supply in blue, return in red); only the computing load differs, 30, 20 and 10 kW today. The dashed arrow is the change studied: moving work from the heavily loaded building toward equal loads. The plant rejects heat to condenser water at 85 °F and draws electricity P_{ch}.

The model. Figure 2 shows the equations along one rack's heat path. The unknowns are the temperatures of every rack, aisle, room and building, the power every rack draws and the heat flow on every edge; the relations are a heat balance at every node and a conductance law on every edge, stated in Section 3. The plant is evaluated on the solved total rack power. The assumptions are these.

- Steady state at every setting; the sweep compares operating points and does not simulate a migration.
- Temperatures in kelvin, power in watts; a rack's power is linear in its temperature with one coefficient for every rack.
- The racks of a building carry equal base loads; balancing changes only how the campus base load is split.
- A rack's heat enters its hot aisle and is driven by the rack's temperature above its cold aisle; recirculation is a conductance proportional to the supply conductance.
- The network's chilled-water boundary is held at 289.26 K, the plant's design entering water, at every load; in steady state all rack heat leaves through it, so the plant's duty equals the total rack power.
- The plant sees condenser water at 85 °F and chilled water entering at 61 °F, the same water as the network's boundary.
- No distribution, pumping, fan or power-conversion losses are counted; PUE is computing plus chiller power over computing power.

The questions. (1) Does balancing the load across the buildings change campus PUE? (2) Does it change the temperature of the hottest rack, and by how much? (3) Does the answer to the first question depend on how the plant is modeled?

Table 1: Every input of the study. Only the split of the base load changes across the sweep; the buildings, the network conductances and the plant are fixed; the network's chilled-water boundary is the water that enters the plant's units.

quantity	value
Buildings	3, identical
Racks per building	30 (3 rooms, one row of 10 racks each)
Base load today, A / B / C	30 / 20 / 10 kW
Leakage coefficient κ, reference T_{ref}	1 W/K per rack, 300 K
Rack conductance G_{rk}	83.33 W/K
Row supply and return conductances G_s, G_{ret}	833.33 W/K, 833.33 W/K
Recirculation ratio μ	1.5
Room and building conductances G_{room}, G_{bldg}	3,333.33 W/K, 10,000 W/K
Conductance sizing: design rack, rack rise, coil approaches	1,000 W, 12 K, 3 K
Chilled-water boundary of the network T_{chw}	289.26 K (61 °F)
Plant	2 water-cooled positive-displacement units of 40 kW
Staging threshold	90 % of the running units' capacity
Condenser water entering, floor	85 °F (302.6 K), 55 °F
Chilled water entering, leaving the evaporator	61 °F (289.26 K), 51 °F
Balancing settings	51, $\alpha = 0, 0.02, \ldots, 1$
Invariance tolerance τ	0.001 in PUE

3 Methods

3.1 The campus thermal network

Table 4 in the appendix collects the symbols. Rack i in row w of room r in building b has a temperature T_i, draws power P_i and passes heat Q_i to its hot aisle; the row has a cold-aisle temperature $T_{c,w}$ and a hot-aisle temperature $T_{h,w}$, the room a temperature T_r and the building T_b. The chilled-water temperature T_{chw} is fixed at 289.26 K. Each law is a residual that vanishes when it holds: the rack's temperature-dependent power, its conductance law, the rack's own heat balance, and the heat balances of the cold aisle, the hot aisle, the room and the building,

$$r_{P,i} = P_i - P_i^0 - \kappa(T_i - T_{\text{ref}}), \tag{1}$$

$$r_{Q,i} = Q_i - G_{rk}(T_i - T_{c,w}), \tag{2}$$

$$r_i = P_i - Q_i, \tag{3}$$

$$r_{c,w} = G_s(T_r - T_{c,w}) + \mu G_s(T_{h,w} - T_{c,w}), \tag{4}$$

$$r_{h,w} = \sum_{i \in w} Q_i - \mu G_s(T_{h,w} - T_{c,w}) - G_{\text{ret}}(T_{h,w} - T_r), \tag{5}$$

$$r_r = G_{\text{ret}}(T_{h,w} - T_r) - G_s(T_r - T_{c,w}) - G_{\text{room}}(T_r - T_b), \tag{6}$$

$$r_b = \sum_{r \in b} G_{\text{room}}(T_r - T_b) - G_{\text{bldg}}(T_b - T_{\text{chw}}), \tag{7}$$

where P_i^0 is the rack's base load, $\kappa = 1$ W/K the leakage coefficient about $T_{\text{ref}} = 300$ K, $G_{rk} = 83.33$ W/K the rack conductance, $G_s = 833.33$ W/K the supply conductance between room and cold aisle, $\mu = 1.5$ the recirculation ratio, $G_{\text{ret}} = 833.33$ W/K the return conductance, and G_{room} and G_{bldg} equal to 3,333.33 and 10,000 W/K. The conductances follow one sizing rule: each link carries the design heat of the heaviest rack, building A's 1,000 W today, at a stated temperature difference, a 12 K rise across the rack's air stream (G_{rk} for one rack, G_s and G_{ret} for the row's 10) and 3

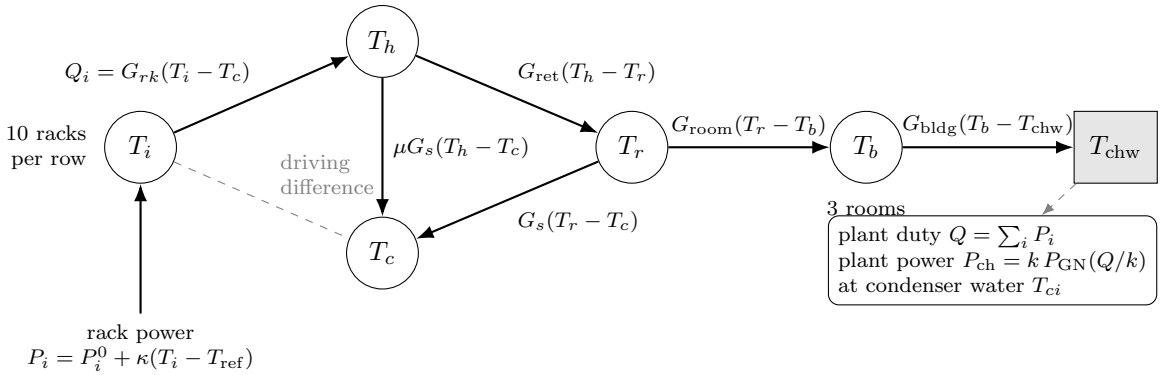


Figure 2: The model of one rack's heat path. Unknowns are temperatures at the nodes (rack T_i, cold aisle T_c, hot aisle T_h, room T_r, building T_b) and heat flows on the edges; the chilled-water temperature T_{chw} is fixed at the plant's design entering water. A rack's power rises with its temperature and enters the rack node; its heat reaches the hot aisle, driven by the rack's temperature above the cold aisle (dashed). Part of the hot aisle's heat recirculates to the cold aisle through the conductance μG_s; the rest returns to the room, which passes it to the building and on to chilled water. Every node balances the heat flows on its edges. The plant is not part of the network: it prices the total rack power at a fixed condenser temperature.

K approaches at the room and building coils (G_{room}, G_{bldg}). Each room holds one row, so the room balance (6) carries that row's return and supply. The cold-aisle balance (4) places the cold aisle between the room and the hot aisle, weighted by the recirculation ratio; this is how poor containment raises the air the racks draw. With every input given, the residuals form a square system, solved by Newton's method from fixed initial temperatures; this is the iteration (9) of the next subsection.

3.2 The network model

The residuals above are all of one kind, and the engine treats them as such; this subsection states the general model they belong to. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{8}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{9}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in this report is estimated from data: every input is stated, and the results are the solutions of these relations and quantities computed from them. Everything that follows is a choice of unknowns and relations on this graph.

3.3 The reduced chain

Because every rack of a building carries the same base load and the rooms of a building are identical, the network reduces to a series chain per rack, and the rack's temperature above chilled water is its power times one resistance R. The cold-aisle balance (4) puts the cold aisle a fraction $\mu/(1+\mu)$ of the way from room to hot aisle; the hot-aisle balance (5) carries n_w racks' heat through the return and recirculation conductances; the room carries n_w racks and the building n_b. Substituting the rack power law (1) gives

$$R = \frac{1}{G_{rk}} + \frac{\mu}{1+\mu}\,\frac{n_w}{G_{\mathrm{ret}} + \mu G_s/(1+\mu)} + \frac{n_w}{G_{\mathrm{room}}} + \frac{n_b}{G_{\mathrm{bldg}}}, \tag{10}$$

$$T_i = T_{\mathrm{chw}} + R\,P_i, \tag{11}$$

$$P_i = \frac{P_i^0 + \kappa(T_{\mathrm{chw}} - T_{\mathrm{ref}})}{1 - \kappa R}, \tag{12}$$

$$P_{\mathrm{IT}} = \frac{\sum_b L_b + N\kappa(T_{\mathrm{chw}} - T_{\mathrm{ref}})}{1 - \kappa R}, \tag{13}$$

with $n_w = 10$ racks per row, $n_b = 30$ per building, $N = 90$ on the campus, $L_b = n_b P_i^0$ the base load of building b, and P_{IT} the campus computing power. Here $R = 0.0225$ K/W, and the leakage amplifies every rack's load by $1/(1-\kappa R) = 1.0230$. Equation (13) is the mechanism behind the first question: every building's computing power is the same affine function of its base load, so the campus total depends on the base loads only through their sum. The reduced chain is not used to compute the results; it checks them, and it agrees with the Newton solution of the full network to below 10^{-9} K in every rack temperature at every setting.

3.4 Campus power and PUE

In steady state all the heat the racks dissipate leaves the campus through chilled water, so the plant's duty is the campus computing power, and PUE follows from the plant's power at that duty,

$$Q = P_{\mathrm{IT}}, \tag{14}$$

$$P_{\mathrm{IT}} = \sum_i P_i, \tag{15}$$

$$\mathrm{PUE} = \frac{P_{\mathrm{IT}} + P_{\mathrm{ch}}(Q)}{P_{\mathrm{IT}}}. \tag{16}$$

3.5 The chiller plant

Each unit follows the Gordon–Ng universal chiller law, a first- and second-law balance of the refrigeration cycle that is linear in three loss coefficients: the internal entropy generation b_1, the equivalent heat leak b_2 and the total heat-exchanger resistance b_3. With T_e the chilled water entering the evaporator, T_{ci} the condenser water entering and Q the unit's load, the law and its explicit form for compressor power are

$$\frac{T_e}{T_{ci}}\Big(1 + \frac{1}{\mathrm{COP}}\Big) = 1 + \frac{b_1 T_e}{Q} + \frac{b_2(T_{ci} - T_e)}{T_{ci}\,Q} + \frac{b_3\,Q}{T_{ci}}\Big(1 + \frac{1}{\mathrm{COP}}\Big), \tag{17}$$

$$P_{\mathrm{GN}}(Q) = \frac{T_{ci}\,Q + b_1 T_e T_{ci} + b_2(T_{ci} - T_e)}{T_e - b_3\,Q} - Q, \tag{18}$$

$$T_{ci} = \max(T_{\mathrm{sink}}, T_{\mathrm{floor}}), \tag{19}$$

where COP is the unit's coefficient of performance, Q/P_{GN}, T_{sink} the condenser water supplied, here 85 °F (302.59 K), and T_{floor} the head-pressure control floor of 55 °F. With all three coefficients zero (18) is the Carnot limit; b_3 makes efficiency peak at part load. Below a minimum stable load $Q_{\min}$, 10 % of capacity, a unit cycles and its time-averaged power is $(Q/Q_{\min})\,P_{\mathrm{GN}}(Q_{\min})$; no setting of this study reaches that branch. Chilled water enters at $T_e = 289.26$ K (61 °F) at every load, the network's boundary temperature.

The shape of the law, the ratios between the three coefficients, is taken from a calibration of (17) on the steady states of the ASHRAE RP-1043 test chiller, a 90-ton machine with $b_1^{\mathrm{ref}} = 78.4267$ W/K, $b_2^{\mathrm{ref}} = 121{,}971$ W and $b_3^{\mathrm{ref}} = 7.711 \times 10^{-5}$ K/W. The coefficients are scaled to the unit's capacity C_u by similarity (entropy generation and heat leak grow with capacity, exchanger resistance falls with it) and multiplied together by one loss factor s, which moves the efficiency level and keeps the shape,

$$b_1 = s\, b_1^{\mathrm{ref}}\, C_u / C_{\mathrm{ref}}, \tag{20}$$

$$b_2 = s\, b_2^{\mathrm{ref}}\, C_u / C_{\mathrm{ref}}, \tag{21}$$

$$b_3 = s\, b_3^{\mathrm{ref}}\, C_{\mathrm{ref}} / C_u, \tag{22}$$

$$\mathrm{IPLV} = 0.01\,\mathrm{COP}_{100} + 0.42\,\mathrm{COP}_{75} + 0.45\,\mathrm{COP}_{50} + 0.12\,\mathrm{COP}_{25}, \tag{23}$$

$$s = \min(s_{\mathrm{FL}}, s_{\mathrm{IPLV}}). \tag{24}$$

ASHRAE 90.1 (ASHRAE, 2019), Table 6.8.1-3, Path A, requires a water-cooled positive-displacement chiller below 75 tons to meet both 0.75 kW/ton at full load (a COP of 4.689) and an integrated part-load value of 0.60 kW/ton (a COP of 5.861). The full-load COP is evaluated at the AHRI 550/590 rating point, chilled water entering at 54 °F and condenser water at 85 °F. The integrated part-load value (23) weighs the COP at 100, 75, 50, 25 % load, with condenser water at 85 °F, 75 °F, 65 °F, 65 °F and chilled water leaving at 44 °F under constant evaporator flow, each point with the condenser floor applied. Each requirement alone fixes the factor at which the unit just meets it, s_{FL} or s_{IPLV} (COP falls monotonically as s grows, so bisection finds each), and the unit meets both at the smaller factor (24). For this unit the IPLV requirement binds: $s = 0.597$, giving $b_1 = 5.921$ W/K, $b_2 = 9{,}209$ W and $b_3 = 3.645 \times 10^{-4}$ K/W, an integrated part-load value of exactly the minimum 5.861 and a full-load COP of 5.577, above the full-load minimum. On the warmer operating water, 61 °F entering, the same unit reaches a full-load COP of 6.251.

The plant runs k of its $n_u = 2$ units, sharing the duty equally, and starts a unit when the duty passes 90 % of the running capacity,

$$k = \min\left(n_u, \lceil Q/(0.9\,C_u) \rceil\right), \tag{25}$$

$$P_{\mathrm{ch}}(Q) = k\, P_{\mathrm{GN}}(Q/k). \tag{26}$$

Because the condenser and chilled-water temperatures are fixed, the plant's power (26) is a function of its duty alone.

3.6 The balancing sweep

A balancing fraction α moves each building's base load in a straight line from today's value L_b^0 to the campus mean $\bar{L} = 20$ kW, and every rack of the building carries an equal share,

$$L_b(\alpha) = (1-\alpha)\, L_b^0 + \alpha\, \bar{L}, \tag{27}$$

$$P_i^0 = L_b(\alpha)/n_b. \tag{28}$$

The sum of the base loads is 60 kW at every α. The campus is rebuilt and solved at 51 evenly spaced settings from $\alpha = 0$ (today) to $\alpha = 1$ (equal loads), one Newton solve each; at each setting the run records each building's base load and computing power, the temperature of its hottest rack, the campus computing power, the plant power, the running units and PUE.

3.7 Survival across plant modeling choices

The plant rests on choices a reader could make differently. Five are varied, each over registered levels: the loss scale, a multiplier of 0.8, 1 or 1.2 on the anchored unit's three coefficients; the staging rule, threshold staging (25) or least-power staging, which runs the number of units that draws least among those that can carry the duty, ties going to fewer units; the unit count, n_u units of 40 kW or $2n_u$ units of half the size, each re-anchored to the code minimum for its own size; the condenser floor, 55 °F or 65 °F, with the unit re-anchored under it; and the chilled-water model, entering at a fixed temperature or falling with load under constant evaporator flow from 51 °F leaving. All 48 combinations are evaluated. In each, the computing power solved at every setting is held fixed and only the plant that prices it changes, and the verdict is scored by its margin,

$$g = \tau - \max_{\alpha} \left|\mathrm{PUE}(\alpha) - \mathrm{PUE}(0)\right|, \tag{29}$$

with the tolerance $\tau = 0.001$. The invariance holds in a combination while $g \geq 0$ and flips where g changes sign from the registered plant's. The joint worst combination is the one with the smallest margin; exact ties go to the smallest level indices in the factor order above, so a tie is reported by that order rather than by round-off.

3.8 Metrics

The first question is answered by campus PUE (16), dimensionless, the standard measure of the electricity a site spends beyond its computing, and by its largest change across the sweep, read against the tolerance $\tau = 0.001$: a change below the tolerance is no change an operator could bill or measure. Because PUE is a ratio, the total computing power (15) and its largest change across the sweep, in kilowatts, are reported beside it; the reduced chain (13) predicts a change of zero, so the reference is floating-point round-off. The second question is answered by the temperature of the hottest rack in each building and on the campus, in kelvin, the quantity that sets how close the hardware runs to its limits, and by its drop from today to the best setting; the reference is the drop the reduced chain (11)–(12) predicts. The third question is answered by the margin g (29) in every plant combination and the count of combinations in which it stays non-negative, read against the tolerance itself, which is the margin a perfectly invariant PUE earns. Cost is counted in steady solves rather than timed.

4 Results

Building loads. Figure 3 plots, against the balancing fraction, each building's base load (dashed) and the computing power its racks draw (solid). The base loads converge on the 20 kW mean by construction (27). The computing power lies above them because hot racks leak: building A draws 30.36 kW today, building B 20.13 kW and building C 9.90 kW, and at equal loads each draws 20.13 kW. The campus total is 60.39 kW at today's split, against 60.39 kW from the reduced chain (13), and its largest change across the 51 settings is below 10^{-9} kW: the 10.23 kW that building A sheds is exactly the power that buildings B and C gain.

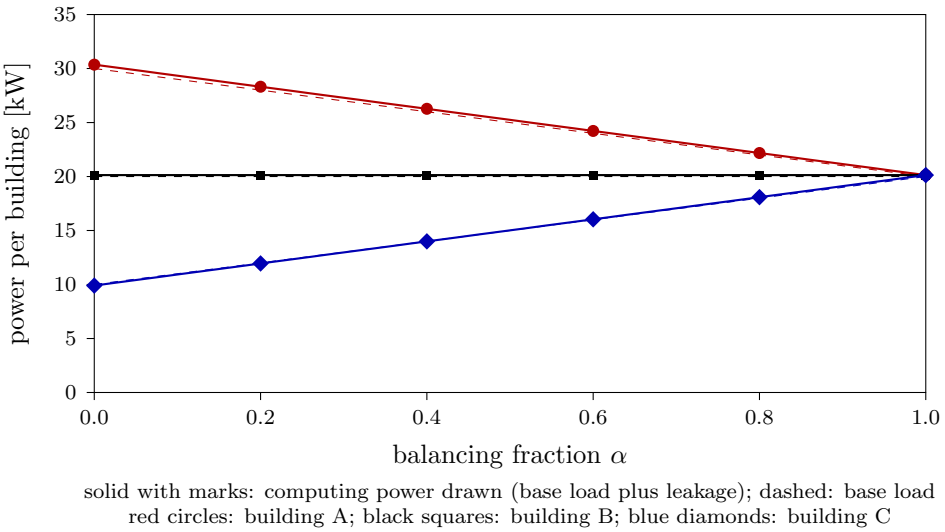


Figure 3: Power per building across the 51 balancing settings, with marks every tenth setting. Dashed lines are the base loads the balancing rule sets; solid lines are the computing power the racks actually draw once their temperature-dependent leakage is included. Building A draws 30.36 kW today for a 30 kW base load; at equal loads every building draws 20.13 kW. The three solid lines are straight and their sum is the same at every setting.

Rack temperatures. Figure 4 plots the temperature of the hottest rack in each building against the balancing fraction, with the campus's hottest rack as a wide gray band. Building A's racks are the hottest on the campus at every setting. Its hottest rack falls in a straight line from 312.0 K today to 304.4 K at equal loads, a drop of 7.67 K, against 7.67 K predicted by (11) and (12). Building C's hottest rack rises by 7.67 K, the same amount, and building B's stays at 304.4 K because its base load is the mean at every setting. The best of the 51 settings is $\alpha = 1.0$: the campus's hottest rack falls monotonically with α, so the best setting is the end of the range. The absolute temperatures follow the conductance sizing rule, not a calibrated server; the drop follows from the network's resistance and the load moved.

Four checks test that the solved network is physically plausible, over all 4590 rack values of the 51 settings. The racks run between 296.69 and 312.03 K, below a bound of 320 K; the coldest cold aisle is 292.73 K, above a bound of 291.15 K; the temperature-driven part of rack power is at most 1.2 % of the base load, below 10 %; and the heat the buildings reject into the chilled water equals the computing power the plant is charged for to a relative difference below 10^{-9}. Every check passes. The plant is also checked against the range of the chiller law's calibration: in all 48 plants of the survival analysis and at every setting's load, no operating point leaves it, with the chilled water entering between 287.90 and 289.26 K.

Campus PUE. Figure 5 plots campus PUE against the balancing fraction inside a band of today's value plus and minus the tolerance, and Table 2 lists the electricity at three settings. PUE is 1.1638 at every setting; its largest change across the sweep is below 10^{-9}, against a tolerance of 0.001. The plant carries 60.39 kW, 75.5 % of its rated capacity, on 2 units of 40 kW each loaded to 75.5 %, draws 9.89 kW at a COP of 6.106, and does the same at every setting. The mechanism is the one (13) and (26) state: with a fixed heat sink the plant's power depends only on its duty, the duty is the campus computing power, and moving work between identical buildings leaves that

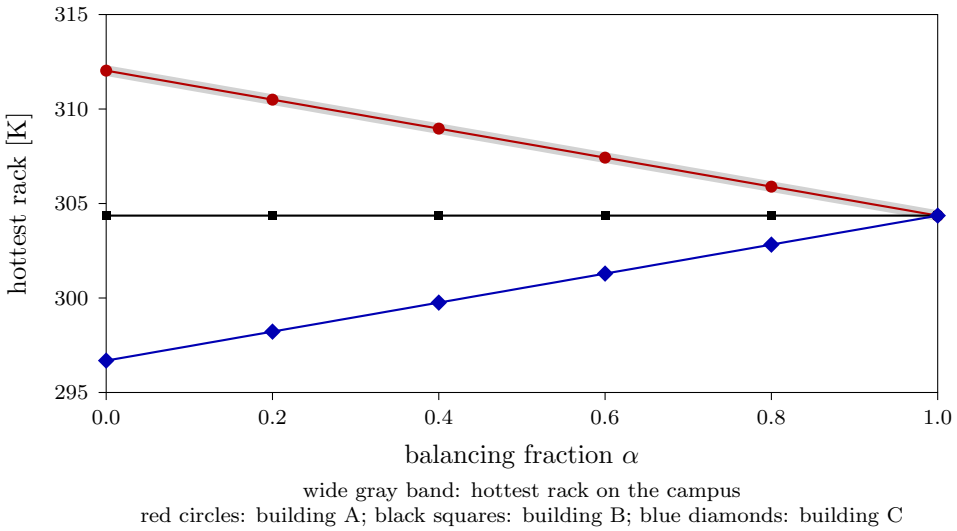


Figure 4: Temperature of the hottest rack in each building across the 51 balancing settings, with marks every tenth setting; the wide gray band under the building A line is the hottest rack on the whole campus. Building A's hottest rack falls from 312.0 K to 304.4 K, building C's rises from 296.7 K by the same amount, and building B's, whose base load is the campus mean at every setting, stays at 304.4 K.

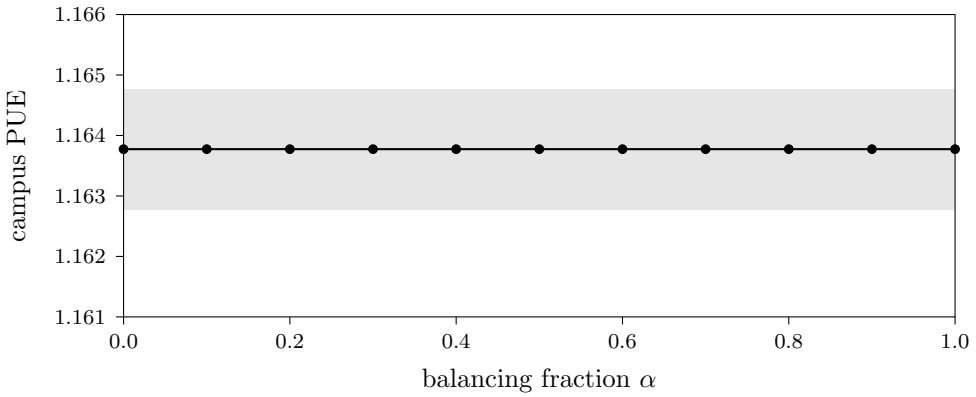
power unchanged, so the plant sees nothing change and PUE cannot move.

Table 2: Campus electricity at today's split ($\alpha = 0$), half way ($\alpha = 0.5$) and at equal loads ($\alpha = 1$): computing power P_{IT}, plant power P_{ch}, their total, the number of chiller units running, the plant's COP (heat removed per unit of electricity) and PUE.

α	P_{IT} [kW]	P_{ch} [kW]	total [kW]	units	COP	PUE
0.0	60.39	9.89	70.28	2	6.106	1.1638
0.5	60.39	9.89	70.28	2	6.106	1.1638
1.0	60.39	9.89	70.28	2	6.106	1.1638

Plant modeling choices. Table 3 scores the invariance under the registered plant, each plant choice changed alone, and the joint worst of all 48 combinations. The invariance holds in 48 of 48: in every combination the margin (29) equals the tolerance to within 10^{-9}, the value a PUE that does not move at all earns. The plant choices do change the level of PUE: across the combinations today's PUE ranges from 1.139 to 1.196; changed alone, the loss scale and the chilled-water model move it, while the staging rule, the unit count and the condenser floor leave it unchanged at this duty. Because every margin is equal, the joint worst combination is chosen by the tie rule and is the first in factor order. The invariance does not come from a particular chiller model but from the computing power the plant is asked to remove.

Cost. 51 steady Newton solves of the campus network, and 2,448 plant evaluations for the survival table, produce every number in this report.



solid line with marks: campus PUE; gray band: today's PUE ± the invariance tolerance of 0.001

Figure 5: Campus PUE across the 51 balancing settings (marks every fifth setting). The gray band spans today's PUE, 1.1638, plus and minus the tolerance of 0.001 within which PUE is called unchanged. The line is flat: its largest departure from today's value is below 10^{-9}, far inside the band.

Table 3: Invariance of campus PUE under the registered plant choices: the registered plant, each choice changed alone, and the joint worst of all 48 combinations. PUE today is the campus PUE at today's split with that plant; the margin g (29) is the tolerance 0.001 minus the largest PUE change across the sweep, and the verdict holds while $g \geq 0$. The invariance holds in 48 of 48 combinations.

plant choice changed	PUE today	margin g	verdict
none (the registered plant)	1.1638	0.001000	holds
loss scale 0.8	1.1395	0.001000	holds
loss scale 1.2	1.1884	0.001000	holds
least-power staging	1.1638	0.001000	holds
4 units of 20 kW	1.1638	0.001000	holds
condenser floor 65 °F	1.1638	0.001000	holds
constant-flow chilled water	1.1707	0.001000	holds
joint worst of 48: loss scale 0.8	1.1395	0.001000	holds

5 Discussion

The invariance of PUE is exact in this model, and it is exact for a reason that also marks where it stops. It needs three things: every relation in the building network is linear, the buildings are identical, and the plant's efficiency depends only on its total duty. Take away any of them and balancing can move PUE. A leakage law convex in temperature, closer to how semiconductor leakage grows, would make the campus total depend on the split and would reward balancing; buildings with different conductances or containment would give each building its own affine function in (13); per-building plants, or distribution and pumping losses that grow with a building's own flow, would make efficiency depend on where the load sits. The result is therefore a property of a load-only plant at a fixed heat sink serving identical, linear buildings, not a general statement that workload placement never affects PUE.

The plant has no weather. Its condenser water is held at the 85 °F full-load rating condition at every setting, while the integrated part-load value the units are anchored to credits part-load operation at 75 °F and 65 °F condenser water; the plant's absolute PUE is therefore pessimistic for

most of a year, although with a fixed heat sink of any temperature the invariance would be the same. Chilled water enters the evaporator at 61 °F, the boundary the building network rejects heat to, which is 7 °F warmer than the rating point at which the units are anchored; the law credits the warmer water with a full-load COP of 6.25 against 5.58 at the rating point. The law's shape is extrapolated from one calibrated centrifugal machine to a small positive-displacement unit by similarity scaling, which that single calibration cannot test; staged unloading, short cycling and the time a second unit takes to start are not represented.

Every setting is a steady state; a real migration passes through transients, and it moves network and storage load as well as computing, which this model does not carry. The leakage coefficient of 1 W/K per rack is a stated value and the conductances follow a design rule, a 12 K rise and 3 K approaches at the heaviest rack, not a calibration to a server or a room, so the absolute rack temperatures, up to 312.0 K, should be read as the temperatures of lumped rack nodes rather than of server inlets or components. The 7.7 K drop is the more transferable number: it is the network's resistance times the load moved, amplified by leakage, and it scales with both. Finally, the best balancing fraction lies at the end of the range because the hottest rack is linear in the load split and building A stays the hottest; with non-identical buildings the best split need not be equal loads.

An earlier version of this study used hand-set conductances ($G_{rk} = 50$, $G_s = G_{\text{ret}} = 100$, $G_{\text{room}} = 1000$ and $G_{\text{bldg}} = 5000$ W/K), a 290 K boundary uncoupled from a plant priced at 54 °F entering water, and rack nodes up to 369 K. It reported a PUE of 1.1821, 63.79 kW of computing power and a hottest rack falling by 26.44 K, from 368.54 K to 342.09 K. The weak conductances put a large resistance between each rack and the chilled water, so the same moved load produced a drop about 3.4 times larger; with conductances sized to the racks' heat and the boundary at the plant's water the invariance of PUE, its survival across the plant choices and the best balancing fraction are unchanged, and only the size of the temperature benefit shrinks. The level of PUE falls to 1.1638 mainly because the plant now runs on the warmer water the buildings reject their heat to.

6 Conclusion

This report asked what happens when the computing load of a three-building data-center campus, today 30, 20 and 10 kW on one shared chilled-water plant, is evened out between identical buildings. It asked first whether campus PUE changes: it does not. PUE stays at 1.1638 across 51 settings from today's split to equal loads, changing by less than 10^{-9} against a tolerance of 0.001, because total computing power, 60.39 kW, depends only on the total load when the buildings are identical and their thermal networks linear, and a plant at a fixed heat sink prices only its total duty. It asked second whether the hottest rack changes: it does, falling by 7.7 K, from 312.0 K to 304.4 K, with the best result at fully equal loads, because the heavily loaded building sheds load; every rack stays within the physical checks at every setting. It asked third whether the PUE result depends on how the plant is modeled: it held in all 48 combinations of loss scale, staging rule, unit count, condenser floor and chilled-water model, even though those choices move the level of PUE. On this campus, balancing the workload is a way to cool the hottest racks, not to lower the electricity bill. The invariance would weaken with non-identical buildings, a convex leakage law, or plants whose efficiency depends on where the load sits; a model with weather at the condenser, per-building distribution losses and a calibrated leakage law is the next step to find how far.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level0_site/workload_migration
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
```

`run.py` solves the campus at every setting, evaluates the plant combinations and writes `results.json`; `make_report.py` reads only `results.json` and writes `report.tex`, and with `--pdf` builds `report.pdf`. The study is deterministic: there are no seeds. The trailing digits of the round-off quantities depend on the linear-algebra kernel, which is why the report states them as bounds.

B Data and problems

The campus is the study's own construction and uses no external data. The plant draws on published values carried in the chiller-plant plugin: the AHRI 550/590 rating temperatures, the ASHRAE 90.1-2019 Table 6.8.1-3 Path A minimums (ASHRAE, 2019), and the Gordon–Ng coefficients calibrated on the ASHRAE RP-1043 chiller data, whose extract is committed with the plugin. The study leaves no corpus problems.

C Notation

Table 4: Notation.

symbol	meaning	unit
α	balancing fraction, 0 today and 1 at equal loads	–
L_b, $\bar{L}$	base load of building b, campus mean	kW
P_i^0, P_i	base load and power drawn by rack i	W
κ, T_{ref}	leakage coefficient and its reference temperature	W/K, K
T_i, T_c, T_h, T_r, T_b	rack, cold-aisle, hot-aisle, room and building temperatures	K
T_{chw}	chilled-water boundary of the network	K
Q_i	heat flow from rack i to its hot aisle	W
G_{rk}, G_s, G_{ret}, G_{room}, G_{bldg}	conductances	W/K
μ	recirculation ratio, recirculation over supply conductance	–
n_w, n_b, N	racks per row, per building, on the campus	–
R	thermal resistance from a rack to chilled water	K/W
P_{IT}, P_{ch}	computing power and plant power	W
Q, C_u, n_u, k	plant duty, unit capacity, installed and running units	W, W, –, –
T_e, T_{ci}	evaporator and condenser water entering	K
b_1, b_2, b_3	entropy generation, heat leak, heat-exchanger resistance	W/K, W, K/W
s	loss factor anchoring a unit to the code minimum	–
τ, g	invariance tolerance and survival margin	–

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