Weather-driven power usage effectiveness of a data-center hall from a three-mass thermal model, and recovery of its parameters from telemetry

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Abstract

The power usage effectiveness (PUE) of a data center, total facility power over IT power, moves with the weather, which decides how much cooling an economizer can carry and how efficiently a chiller carries the rest. This study computes that dependence for one hall of about 100 kW of IT, modeled as three heat-storing masses (IT equipment, room air, floor slab), cooled by an air-side economizer and two 100 kW air-cooled chillers at the ASHRAE 90.1 minimum efficiency, and driven by a synthetic week of outdoor temperature (9 to 39 °C) and IT load (80 to 120 kW) at 15-minute steps. It asks what PUE the week produces and why, which forced supply-air setpoint minimizes facility power in cold, mild and hot weather, and whether the parameters can be recovered from the model's own telemetry. The network is integrated by backward differences, the chillers priced by the Gordon–Ng law anchored to the code minimums, and the parameters fitted by least squares. It is a forward study: every input is stated, and the recovery of the parameters is a well-posedness check on noise-free records produced by the same model, not an uncertainty result. The mean PUE is 1.121 (range 1.017 to 1.268), the economizer carrying some of the cooling in 51 % of the week; the swing follows the plant's change from free cooling, at an effective coefficient of performance of 33, to the chiller, at down to 3.5. The hall follows the weather and the supply policy through the week: the room air reaches 31.7 °C and the IT equipment 39.8 °C on the last and hottest afternoon, where the temperature-dependent leakage reaches 2597 W. In all three weathers the best setpoint is the cold floor, 17.9 °C, in all 96 combinations of plant modeling choices, with a smallest margin of 65 W. From noise-free telemetry of the same model the supply policy is recovered exactly, the chiller loss scale within 10^{-6}, the distribution fraction within 10^{-9}, and the IT heat capacity and conductance to within 0.31 %.

1 Introduction

Data centers turn nearly all of the electricity they draw into heat, and the power spent removing that heat is the largest overhead a facility adds to its IT load. The industry summarizes that overhead by the power usage effectiveness, total facility power divided by IT power, where 1.0 would mean no overhead at all. Brady, Kapur, Summers and Thompson (2013) assess critically how the figure is calculated for an operating facility, and a PUE is not a single number without the conditions under which it was obtained. The weather is one of those conditions. Free cooling, which uses cold outdoor air or water in place of mechanical refrigeration whenever the climate allows, is the main lever on the overhead, and Zhang, Shao, Xu, Zou and Tian (2014) review its air-side, water-side and heat-pipe forms. A facility with an economizer therefore has a PUE that

moves hour by hour with the outdoor temperature, and predicting that movement, or choosing a supply-air setpoint against it, needs a model of the particular hall and plant.

Two modeling traditions meet in this problem. On the building side, lumped resistance–capacitance networks describe the heat dynamics of a building with a few heat-storing nodes joined by conductances, and Bacher and Madsen (2011) identify such gray-box networks from measured data, selecting among models of increasing order with likelihood-ratio tests; a heat capacity is identifiable only from records in which temperatures move. On the plant side, the thermodynamic chiller model of Gordon and Ng writes a chiller's efficiency from the first and second laws with three physical loss coefficients (internal entropy generation, heat leak and heat-exchanger resistance), and Chua, Ng and Gordon (1996) examine its fundamental properties experimentally on reciprocating chillers. How efficient a chiller must be is set by building codes: ASHRAE Standard 90.1 (ASHRAE, 2019) fixes a full-load and an integrated part-load minimum for each class and size of chiller, rated at the conditions of AHRI Standard 550/590.

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses its data-center and HVAC plugins to write the hall's three masses as energy balances with storage and their couplings as heat-flow laws, integrates them through a synthetic week, prices the cooling with an economizer and a staged chiller plant, and fits the model's parameters back from the resulting records. Section 3 states the framework and each of these constructs precisely.

This study makes no claim of novelty; it combines established pieces and checks them against each other on one hall. It asks three questions: what PUE a week of weather and load produces and why it swings; which forced supply-air setpoint minimizes total facility power in cold, mild and hot weather, and whether that answer depends on how the chiller plant is modeled; and whether the supply policy, the chiller plant's efficiency level and the IT equipment's heat capacity and conductance can be recovered from the week's telemetry. It is a forward study: every input is stated, and the week, the steady states and the setpoint sweep are solutions of the model's own relations. The third question fits parameters by least squares to telemetry the same model produced without noise, so its errors measure whether the problem is well posed, not how uncertain an estimate on field data would be. Section 2 states the hall, the plant, the data and the model, Section 3 the methods, Section 4 the results, Section 5 their limits and Section 6 the answers; the appendices give the reproduction recipe, the data and the notation (Table 4).

2 System and data

The hall. Figure 1 draws the system. One data-center hall holds about 100 kW of IT equipment. Three parts of it store heat on different time scales: the IT equipment itself (servers and their chassis), the room air, and the concrete floor slab with the structure. The IT equipment dissipates its electrical power as heat into the air; the air exchanges heat with the slab and, through the building envelope, with the outdoors; the cooling plant removes heat from the air by delivering supply air at a set temperature. IT power is the compute load plus a temperature-dependent silicon leakage (extra power drawn by warm chips) plus the server fans, which speed up when the supply air is warm. Table 1 lists the hall's constants. The quantity of interest is the PUE, total facility power over IT power.

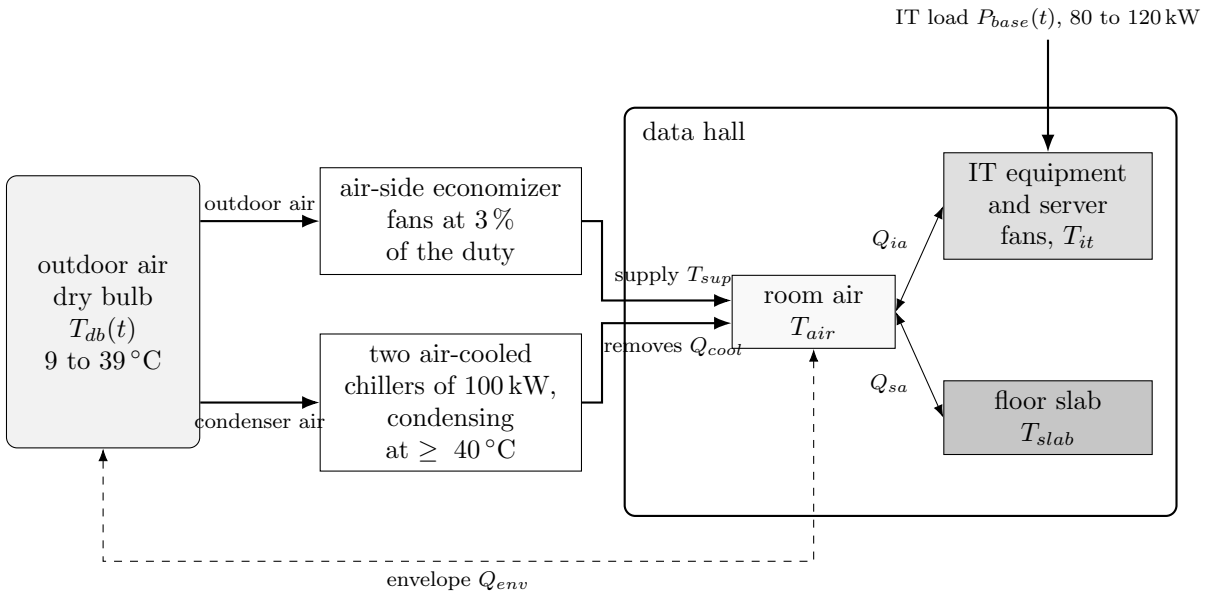


Figure 1: The system. One data-center hall of about 100 kW of IT stores heat in three places: the IT equipment with its server fans, the room air and the floor slab. An air-side economizer cools the hall with outdoor air while the outdoor temperature allows; above a crossover temperature a staged plant of air-cooled chillers, rejecting heat to the same outdoor air, carries the cooling duty Q_{cool} instead. Heat also crosses the building envelope. Facility power is the IT power plus the cooling power, and the power usage effectiveness is their ratio to the IT power.

The plant. Cooling is an air-side economizer integrated with a chiller plant. The supply-air temperature follows the outdoor dry bulb (the air temperature an ordinary thermometer reads) plus 3 K, between a floor of 17.9 and a cap of 26.9 °C. Below the crossover, 23.9 °C outdoors, the outdoor air carries all of the cooling for an economizer fan power of 3 % of the duty; over a blend band of 1 K above it the chiller share rises to the whole duty. The chiller share is carried by two staged air-cooled units of 100 kW each, the least efficient machine of that size meeting both ASHRAE 90.1 minimums for air-cooled chillers, a full-load coefficient of performance (COP, cooling delivered per unit of electrical power) of 2.96 and an integrated part-load value of 4.02. Each unit rejects its heat to the outdoor air, with head-pressure control holding the condensing temperature at or above 40 °C, and receives chilled water at the 12.2 °C (54 °F) entering temperature of the AHRI rating.

The data. Everything is synthetic and deterministic; no field data are used. The week runs 168 hours at 15-minute steps, 673 samples in all, with no measurement noise; the random seed 0 is set but nothing is drawn. With t in hours, the outdoor dry bulb T_{db} (K), the relative humidity ϕ and the IT compute load P_{base} (W) are

$$T_{db}(t) = 293 + 11 \sin\big(2\pi(t-9)/24\big) + 9\,t/168, \tag{1}$$

$$\phi(t) = 0.55 - 0.25 \sin\big(2\pi(t-9)/24\big), \tag{2}$$

$$P_{base}(t) = 100000 + 20000 \sin\big(2\pi(t-13)/24\big), \tag{3}$$

with the humidity clipped to [0.1, 0.95]. The dry bulb spans 9.0 to 39.4 °C: a daily swing of ±11 K around 20 °C on a warming trend of 9 K over the week. The humidity enters only the wet bulb,

Table 1: Constants of the hall. The heat capacities and conductances define the three-mass network; the leakage and fan constants define the temperature-dependent part of the IT power.

quantity	symbol	value	unit
heat capacity of the IT equipment	C_{it}	1.0×10^7	J/K
heat capacity of the room air	C_{air}	2.0×10^6	J/K
heat capacity of the slab	C_{slab}	3.0×10^8	J/K
IT-to-air conductance	UA_{ia}	1.5×10^4	W/K
slab-to-air conductance	UA_{sa}	5.0×10^3	W/K
cooling conductance	G	2.5×10^4	W/K
envelope conductance	UA_{env}	500	W/K
leakage coefficient	α	200	W/K
leakage reference temperature	T_{ref}	300	K
server-fan base power	P_{f0}	5000	W
server-fan ramp	k_f	0.05	1/K
server-fan reference temperature	T_f	293	K

7.0 to 25.5 °C by Stull's approximation, which this air-cooled plant does not use. The records the identification reads are the week's own simulated series: outdoor dry bulb, supply temperature, cooling duty and cooling power, IT load and leakage, and the IT and air temperatures.

The model. Figure 2 shows the unknowns and the relations that tie them. The unknowns are the three temperatures and the heat flows on the four couplings; each mass contributes an energy balance with a storage term and each coupling a linear heat-flow law. Section 3 states the relations.

Assumptions.

- SI units: temperatures in kelvin in the equations and in degrees Celsius in the text and figures, power in watts.
- Each mass is one well-mixed lump; heat flows are linear in temperature differences; there is no solar gain.
- Every driver of the week varies in time and is evaluated at each step: the outdoor temperature seen by the network spans 9.0 to 39.4 °C with the dry bulb, and the supply temperature moves between the policy's floor of 17.9 and its cap of 26.9 °C; the same weather sets the plant's regime and efficiency, and the same supply temperature the fan power.
- The plant is evaluated on the solved duty and does not feed back into the hall; its capacity is never reached.
- Chilled water enters every unit at the rating temperature; the supply setpoint does not change the chiller's efficiency.
- The telemetry is noise-free and generated by the model being identified.

The questions. (1) What PUE does the week produce, and why does it swing? (2) Which forced supply setpoint minimizes total facility power in cold, mild and hot weather, and does the answer survive the choices made in modeling the chiller plant? (3) Can the supply policy, the chiller plant's loss scale and distribution fraction, and the IT equipment's heat capacity and conductance be recovered from the week's telemetry?

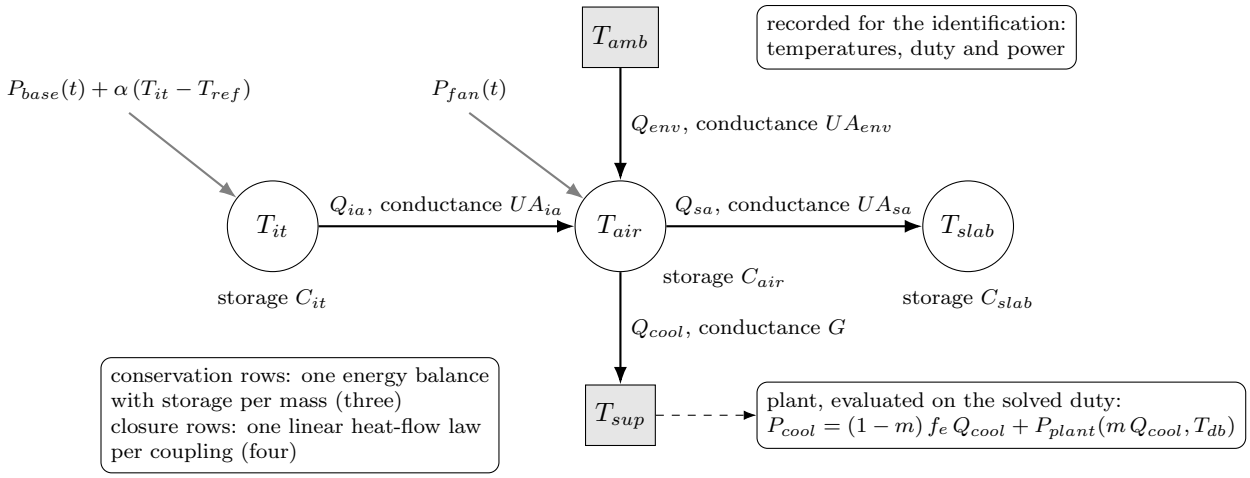


Figure 2: The model. The unknowns are the temperatures of the three masses (circles) and the heat flows between them; the outdoor and supply temperatures (squares) are boundary values. Each mass contributes one energy balance with a storage term (a conservation row), each coupling one linear heat-flow law (a closure row); the IT load with its temperature-dependent leakage and the server-fan power enter as sources. The cooling plant is evaluated on the solved duty and the outdoor temperature and does not feed back into the hall. The identification reads the solved series back and fits constants of the model to them; the network itself uses no recorded values.

3 Methods

3.1 The three-mass thermal network

The unknowns are the temperatures T_{it}, T_{air} and T_{slab} (K) of the three masses and the heat flows (W) between them. Each mass stores the energy $C\,T$; its balance equates the rate of change of that energy to the heat flowing in, and each coupling carries a heat flow proportional to the temperature difference across it:

$$C_{it}\,\dot{T}_{it} = P_{base}(t) + \alpha\,(T_{it} - T_{ref}) - Q_{ia}, \tag{4}$$

$$C_{air}\,\dot{T}_{air} = Q_{ia} + Q_{env} + P_{fan}(t) - Q_{sa} - Q_{cool}, \tag{5}$$

$$C_{slab}\,\dot{T}_{slab} = Q_{sa}, \tag{6}$$

$$Q_{ia} = UA_{ia}\,(T_{it} - T_{air}), \tag{7}$$

$$Q_{sa} = UA_{sa}\,(T_{air} - T_{slab}), \tag{8}$$

$$Q_{env} = UA_{env}\,(T_{amb} - T_{air}), \tag{9}$$

$$Q_{cool} = G\,(T_{air} - T_{sup}), \tag{10}$$

$$P_{fan} = P_{f0}\left(1 + k_f \max(0,\, T_{sup} - T_f)\right), \tag{11}$$

with the constants of Table 1. The balances (4)–(6) are the conservation rows of the network and (7)–(10) its closure rows. The three masses respond on well-separated time scales: the air in about $C_{air}/(UA_{ia} + UA_{sa} + G + UA_{env})$, 44 s; the IT equipment in C_{it}/UA_{ia}, 11 min; and the slab in C_{slab}/UA_{sa}, 16.7 h. The network is integrated over the week by the second-order backward-difference formula (BDF2) at a fixed step Δt of 900 s, the first step by backward Euler; at each

step the energies $E = C\,T$ at the new time solve

$$\frac{3E^{n+1} - 4E^{n} + E^{n-1}}{2\,\Delta t} = \Phi\big(T^{n+1}, t^{n+1}\big), \tag{12}$$

with Φ the right-hand sides of (4)–(6), by Newton's method to an absolute residual tolerance of 1×10^{-3} W in at most 40 iterations, starting from the steady state of the network under the weather and load at the start of the week. The outdoor temperature T_{amb} and the supply temperature T_{sup} are boundary values of the network, re-evaluated at every step from the dry bulb (1) and from the supply policy (13) of the next subsection, as are the IT load (3) and the fan power (11). Over the week T_{amb} spans the dry bulb's 9.0 to 39.4 °C and T_{sup} the policy's 17.9 to 26.9 °C. The supply policy therefore reaches the hall's temperatures as well as the fan power and the plant's operation.

3.2 Supply policy and plant power

The supply temperature, the crossover, the mechanical share m of the duty and the cooling power are

$$T_{sup} = \min\big(T_{max},\, \max(T_{min},\, T_{db} + a)\big), \tag{13}$$

$$T_{\times} = T_{max} - a, \tag{14}$$

$$m = \min\big(1,\, \max(0,\, (T_{db} - T_{\times})/\Delta_b)\big), \tag{15}$$

$$P_{cool} = (1 - m)\, f_e\, Q_{cool} + P_{plant}(m\, Q_{cool},\, T_{db}) + f_d\, Q_{cool}, \tag{16}$$

with approach $a = 3$ K, $T_{min} = 291$ K, $T_{max} = 300$ K, blend band $\Delta_b = 1$ K, economizer fraction $f_e = 0.03$ and distribution fraction $f_d = 0$. The mechanical share depends on the outdoor temperature alone, through (15), and not on the supply setpoint. The chiller plant stages two identical units of capacity $C_u = 100$ kW; a duty q runs k units sharing it equally, and each unit follows the Gordon–Ng law with its condenser air floored by head-pressure control:

$$k = \min\big(n,\, \lceil q/(0.9\, C_u) \rceil\big), \tag{17}$$

$$P_{plant}(q, T_{db}) = k\, P_u(q/k,\, T_{db}), \tag{18}$$

$$x = q_u / C_u, \tag{19}$$

$$T_{ci} = \max\big(T_{db},\, 313.15 - 16.7\, x\big), \tag{20}$$

$$P_u = \frac{T_{ci}\, q_u + b_1 T_{chi} T_{ci} + b_2 (T_{ci} - T_{chi})}{T_{chi} - b_3\, q_u} - q_u, \tag{21}$$

$$P_u(q_u) = (q_u / q_{min})\, P_u(q_{min}), \tag{22}$$

where (21) holds for $q_u \geq q_{min}$ and (22) below it, where the unit cycles; $q_{min} = 10$ kW, the chilled water enters at $T_{chi} = 285.37$ K at every load, and the floor in (20) holds a saturated condensing temperature of 40 °C through a condenser approach that shrinks with unit load. The loss coefficients b_1 (entropy generation, W/K), b_2 (heat leak, W) and b_3 (heat-exchanger resistance, K/W) keep the ratios fitted to 486 steady states of one 316.5 kW water-cooled centrifugal test chiller, $b_1^{ref} = 78.4267$ W/K, $b_2^{ref} = 1.220 \times 10^{5}$ W and $b_3^{ref} = 7.711 \times 10^{-5}$ K/W, scaled to the unit's capacity

and multiplied by one factor s^*:

$$(b_1, b_2, b_3) = s^*\big(\rho\, b_1^{ref},\ \rho\, b_2^{ref},\ b_3^{ref}/\rho\big), \tag{23}$$

$$\rho = C_u / C_{ref}, \tag{24}$$

$$\mathrm{COP}_{FL}(s_{FL}) = \mathrm{COP}_{FL}^{min}, \tag{25}$$

$$\mathrm{IPLV}(s_{IP}) = \mathrm{COP}_{IPLV}^{min}, \tag{26}$$

$$\mathrm{IPLV} = 0.01\,A + 0.42\,B + 0.45\,C + 0.12\,D, \tag{27}$$

$$s^* = \min(s_{FL},\, s_{IP}), \tag{28}$$

where A, B, C and D are the COPs at 100, 75, 50 and 25 % load with condenser air at 35.0, 26.7, 18.3, 12.8 °C (95, 80, 65 and 55 °F), chilled water leaving at 6.7 °C with a full-load rise of 5.56 K in proportion to load, and the floor (20) applied at each point, as the AHRI 550/590 rating prescribes. Each factor in (25) and (26) is found by bisection, the machine's efficiency falling as the factor grows; the smaller one gives the least efficient machine that meets both minimums. For this unit $s^* = 0.715$, so $b_1 = 17.71$ W/K, $b_2 = 2.755 \times 10^4$ W and $b_3 = 1.745 \times 10^{-4}$ K/W; the unit's full-load COP at the rating is 4.32 against the minimum 2.96, and its IPLV equals the minimum 4.02, so the part-load requirement sets the machine. Finally, the IT power, the PUE and the effective COP of the plant are

$$P_{IT} = P_{base} + \alpha\,(T_{it} - T_{ref}) + P_{fan}, \tag{29}$$

$$\mathrm{PUE} = (P_{IT} + P_{cool})/P_{IT}, \tag{30}$$

$$\mathrm{COP}_{eff} = Q_{cool}/P_{cool}. \tag{31}$$

3.3 The network model

The two preceding subsections write the hall as a network of unknowns and relations: the three energy balances are its conservation relations, each with a storage term, the four heat-flow laws its constitutive relations, and the outdoor and supply temperatures its boundary values. This subsection states the general model they belong to. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{32}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{33}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in the week, the steady states or the setpoint sweep is estimated from data: every input is stated, and those results are the solutions of these relations and quantities computed from them. The identification below runs the other way, fitting seven constants of the model back from records the solved relations produced, by least squares, and is stated on its own terms where it is used. Everything that follows is a choice of unknowns and relations on this graph.

3.4 Steady states and the setpoint sweep

For the setpoint question the network is solved at steady state, every time derivative in (4)–(6) set to zero, with the outdoor temperature at 7.9, 18.9 and 27.9 °C (called cold, mild and hot), a relative humidity of 50 %, and an IT load of 110 kW. The supply temperature is forced to each of 31 setpoints from 17.9 to 32.9 °C in steps of 0.5 K, and the fans follow (11) at the forced setpoint. Each steady state gives the duty Q_{cool} and the IT power (29); the plant prices the duty by (16), with the mechanical share (15) computed from the outdoor temperature, which gives $m = 0$, 0 and 1 in the three weathers. The total

$$P_{tot}(T_s) = P_{IT}(T_s) + P_{cool}(T_s) \tag{34}$$

is compared across setpoints T_s, and the best setpoint is the one with the least total.

3.5 Survival across plant modeling choices

The plant model rests on choices a reader could make differently, so the best setpoint of each weather is re-examined under each registered alternative. This is a sensitivity analysis over modeling choices; it says how far a conclusion travels across the choices listed here and makes no probability statement about them. The duty and the IT power at every setpoint are held; only the plant pricing the duty changes. In each combination the margin of weather w is the smallest extra power any other setpoint costs over the registered plant's best setpoint j_w^*, and the combination's margin is the smallest over the weathers:

$$c_w = \min_{j \neq j_w^*} \left(P_{tot,j} - P_{tot,j_w^*} \right), \tag{35}$$

$$g = \min_w c_w. \tag{36}$$

A negative margin would move a best setpoint. The choices are the loss scale (0.8, 1, 1.2, multiplying the anchored unit's losses), the staging rule (threshold staging as in (17), or the unit count that draws least), the unit count (two units of 100 kW, or four of 50 kW re-anchored at their own size), the head-pressure floor (four variants, each re-anchoring the unit: floor 313.15 K less 16.7 K per unit load; no head-pressure floor; floor 311.75 K less 16.7 K per unit load; fixed condenser floor 296.45 K) and the chilled water (fixed entering temperature, or constant flow with the entering temperature falling with load), $3 \times 2 \times 2 \times 4 \times 2 = 96$ combinations in all. The report lists the registered plant, every combination that changes one choice, and the combination with the smallest margin g, ties going to the first in the order the choices are listed.

3.6 Parameter identification

The week's 673 records are treated as telemetry, and three estimators recover the model from them. The supply policy is read from the plateaus and the floating part of the supply record,

$$\hat{T}_{min} = \min_k T_{sup,k}, \tag{37}$$

$$\hat{T}_{max} = \max_k T_{sup,k}, \tag{38}$$

$$\hat{a} = \frac{1}{|\mathcal{F}|} \sum_{k \in \mathcal{F}} \left(T_{sup,k} - T_{db,k} \right), \tag{39}$$

with $\mathcal{F}$ the 210 samples more than 0.05 K inside both plateaus. The chiller plant's loss scale s, which multiplies the three loss coefficients of the registered unit (1 is that unit), and the distribution

fraction f_d are fitted to the cooling power of the fully economized samples $\mathcal{E}$ and the fully mechanical samples $\mathcal{M}$, whose model powers are

$$\hat{P}_k = (f_d + f_e)\, Q_k, \tag{40}$$

$$\hat{P}_k = f_d\, Q_k + P_{plant}(Q_k,\, T_{db,k};\, s), \tag{41}$$

$$(\hat{s}, \hat{f}_d) = \arg\min_{s, f_d} \sum_{k \in \mathcal{E} \cup \mathcal{M}} \left((P_k - \hat{P}_k)/\hat{P}_k \right)^2, \tag{42}$$

with the first form for $k \in \mathcal{E}$ (323 samples) and the second for $k \in \mathcal{M}$ (329 samples); the 21 blend samples mix the two and are excluded, and f_e is known. Each residual is divided by the model power, recomputed at every iterate, which suits meter noise proportional to the reading. Writing θ for the pair (s, f_d), $\rho(\theta)$ for the vector of those residuals and J for its Jacobian, the fit is Gauss–Newton from $s = 1$ and $f_d = 0$, stopped when both steps fall below 10^{-12} relative:

$$\theta \leftarrow \theta - (J^\top J)^{-1} J^\top \rho(\theta). \tag{43}$$

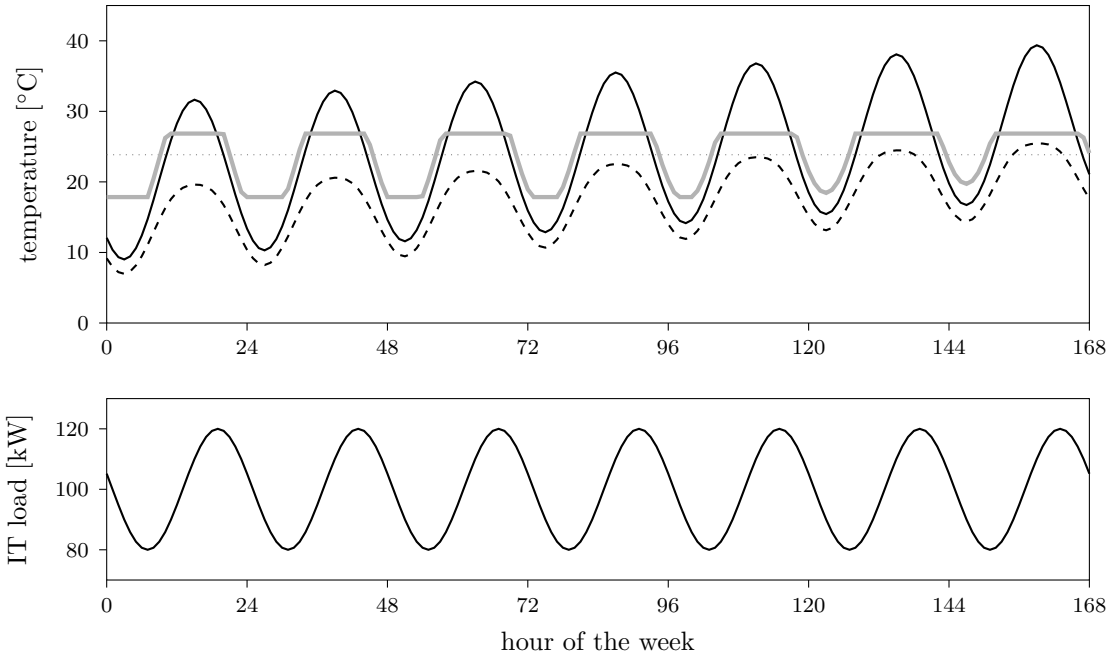
The IT equipment's heat capacity and conductance follow from its balance (4), in which every quantity but C_{it} and UA_{ia} is recorded, by ordinary least squares over forward differences:

$$P_{base,k} + \alpha\,(T_{it,k} - T_{ref}) = C_{it}\, \frac{T_{it,k+1} - T_{it,k}}{\Delta t} + UA_{ia}\,(T_{it,k} - T_{air,k}). \tag{44}$$

The fit uses 672 rows. On a steady record the first regressor vanishes and C_{it} is not identifiable; the transient is what identifies it. Because the telemetry is noise-free and produced by the model being identified, the three fits test whether the problem is well posed, not how the estimators behave on field data.

3.7 Metrics

The first question is answered by the PUE (30): its time-weighted mean over the week, each sample weighted by its step, and its least and greatest values. The reference is 1.0, a facility with no overhead. Why it swings is read from the share of the week in which the economizer carries at least part of the duty ($m < 1$), from the effective COP (31) in each regime, whose reference for free cooling is $1/f_e = 33.3$ and for the chiller the code's full-load minimum of 2.96 at the rating conditions, and from the swing of each temperature, its greatest less its least value in kelvin, read against the time scales above. The energies of the week, in kWh, give the split between IT and cooling. The second question is answered by the best setpoint of (34) in each weather and by the extra power of the warmest setpoint over the best, in kW; the reference is zero, no preference between setpoints. Its robustness is the margin (36) in watts, whose reference is zero: a positive margin in every combination means no registered plant choice moves the best setpoint. The third question is answered by the error of each recovered parameter against the value that generated the telemetry: absolute for the supply policy, the loss scale (truth 1) and the distribution fraction (truth 0), whose reference is round-off because the model that generated the data is the model fitted, and relative for C_{it} and UA_{ia}, whose reference is the error of reading a BDF2 record with a forward difference. How far the chiller law is used beyond its measurements is counted as the share of the running mechanical samples, and of the chiller energy, outside the ranges of chilled-water and condenser temperature, load and lift covered by the chiller law's test data, and as the share of chiller energy at which the head-pressure floor sets the condenser.



top: outdoor dry bulb (solid), wet bulb (dashed), supply temperature from the policy (thick gray), crossover (dotted)
bottom: IT compute load

Figure 3: The drivers of the week, sampled hourly. Top: the outdoor dry bulb, a daily cycle on a warming trend from 9.0 to 39.4 °C; the wet bulb derived from it and the humidity cycle; and the supply temperature the policy assigns, which rests on its floor of 17.9 °C at night and on its cap of 26.9 °C on afternoons above the crossover of 23.9 °C. Bottom: the IT compute load, a daily cycle between 80 and 120 kW peaking mid-afternoon.

4 Results

Weather, load and supply temperature. Figure 3 plots the drivers of the week, hour by hour: the outdoor dry bulb, the wet bulb and the supply temperature the policy assigns in the upper panel, the IT compute load in the lower. The dry bulb crosses the crossover of 23.9 °C on every afternoon, so each day has a free-cooling night and a chiller afternoon, and the warming trend lengthens the afternoons as the week goes on. The supply temperature rests on its floor of 17.9 °C at night, floats with the outdoor air through the morning and sits on its cap of 26.9 °C through the afternoon. The IT load swings between 80 and 120 kW, peaking in the afternoon with the heat.

Temperatures of the hall. Figure 4 plots the three temperatures. The room air swings by 10.4 K over the week and the slab by 8.2 K, while the IT equipment swings by 13.3 K and runs 4.9 to 8.4 K above the air, hour by hour. The supply temperature is what carries the hall: by (10) the room air sits the cooling duty divided by the cooling conductance above the supply air, 2.1 to 5.4 K, so the 9 K the policy travels from its floor to its cap moves the room with it, and the IT equipment rides above the air on the daily cycle of the compute load, 40 kW from trough to peak across the IT-to-air conductance (7). The outdoor air also reaches the room directly through the envelope (9), 4.0 kW inward at the hottest hour of the week against 6.3 kW outward on the coldest night: small beside the 80 to 120 kW the IT equipment puts into the air, but carried inward exactly in the hours when the chiller is least efficient. The slab, with its 16.7 h time scale, damps the daily

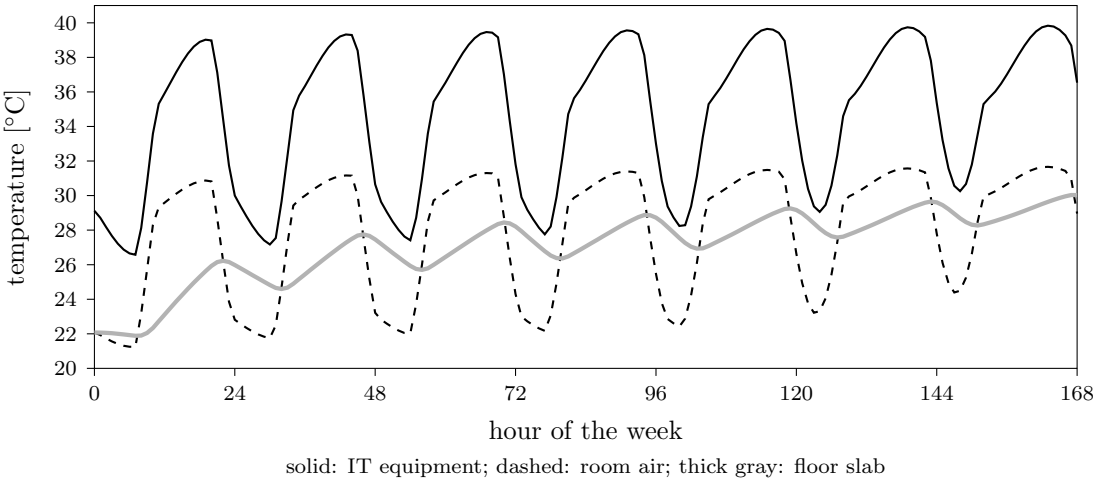


Figure 4: Temperatures of the three masses over the week, sampled hourly. The IT equipment runs between 26.6 and 39.8 °C, the room air between 21.2 and 31.7 °C and the slab between 21.9 and 30.0 °C. All three ride the supply temperature of the policy: the room air sits a few kelvin above it, the IT equipment a few kelvin above the air, and every daily peak falls on the afternoon, when the supply air is on its cap, the outdoor air is warmest and the compute load is highest. The slab, the slowest mass, damps the daily cycle and follows the warming trend of the week.

cycle and follows the warming trend of the week instead. The IT temperature matters to power through the leakage term, which ranges from −55 to 2,597 W, and the supply temperature through the fans, 5,000 to 6,750 W.

Power usage effectiveness. Figure 5 plots the hourly PUE with the chiller hours shaded. The weekly mean is 1.121, against 1.0 for a facility without overhead, and the PUE ranges from 1.017 to 1.268. Of the 673 samples, 323 are fully economized, 21 blended and 329 fully mechanical, so the economizer carries at least part of the duty in 51 % of the week. The swing is the plant's: on free cooling the effective COP (31) is 33.3, which is $1/f_e$ exactly, and a PUE near 1.02; on the chiller it falls to between 3.5 and 5.2, all above the code's full-load minimum of 2.96 at the rating conditions, and the PUE rises toward 1.27 on the hottest afternoon, where the outdoor air is warmest and the load highest. Over the week the IT equipment uses 18,116 kWh and the cooling 2,322 kWh, 20,438 kWh in all, with a peak facility power of 159.9 kW.

Best supply setpoint. Figure 6 plots, for each weather, how much more total facility power (34) each forced setpoint costs than the best one. In all three weathers the best setpoint is the cold floor, 17.9 °C, and power rises steadily as the setpoint warms: running at 32.9 °C costs 6.24 kW more in cold weather, 6.24 kW in mild and 6.17 kW in hot, against a reference of zero. The mechanism is visible in the parts: from the coldest to the warmest setpoint the IT power rises by 6.28 kW, as the warmer hall raises the leakage and the fans ramp, while the duty falls by 1.20 kW as the envelope exchange shifts with the warmer air. The share of the duty the outdoor air carries, (15), is set by the outdoor temperature against the plant's crossover, not by the forced setpoint, so a warmer setpoint saves no chiller energy here; it only adds temperature-driven leakage and fan power, and the small fall in duty is priced at the economizer's 3 % in cold and mild weather and at the chiller's COP in hot weather. In this model the best setpoint is the same in every weather.

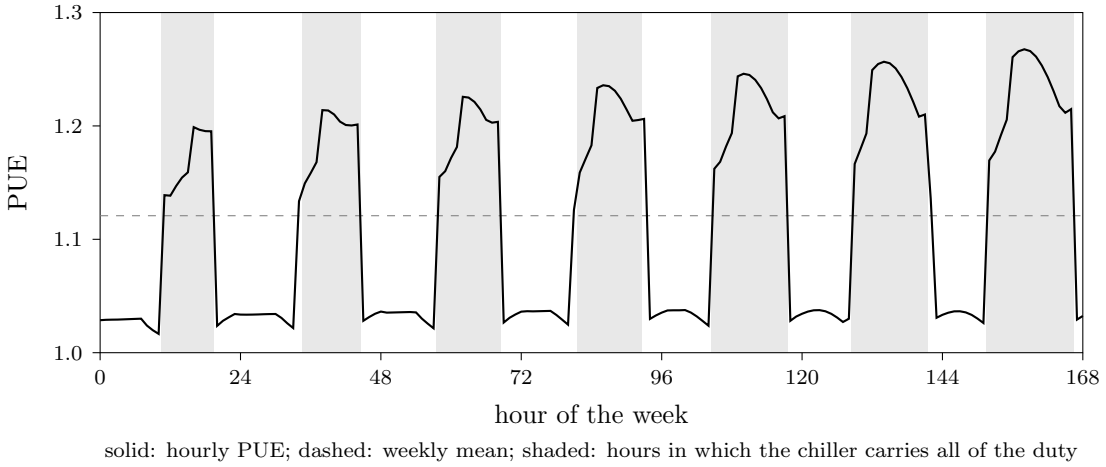
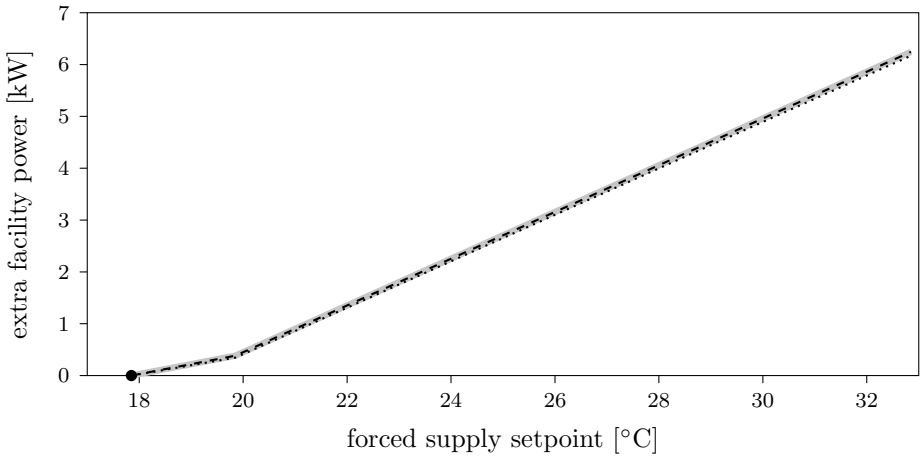


Figure 5: Power usage effectiveness over the week, sampled hourly. Shaded bands mark the chiller hours. The PUE falls to 1.017 when outdoor air carries the cooling and rises to 1.268 on the hottest chiller afternoon, around a weekly mean of 1.121; of the 673 15-minute samples, 323 are fully economized, 21 blended and 329 fully mechanical.

Survival across plant choices. Table 2 lists the margins (35) of the registered plant, of each single change of plant choice, and of the combination closest to moving a best setpoint. For the registered plant the margins are 95.8 W (cold), 95.8 W (mild) and 86.5 W (hot), the step from the floor to the next setpoint 0.5 K warmer. The cold and mild margins are the same in every combination, because no chiller runs in those weathers; the hot margin moves with the plant, and it is smallest, 64.5 W, for loss scale 1.2, four units of 50 kW, no head-pressure floor. The cold floor stays best in every one of the 96 combinations, against a reference of zero: every margin is positive.

Parameter identification. Table 3 compares the recovered parameters with the values that generated the telemetry. The supply approach and the floor and cap of the supply policy are recovered exactly, to below 10^{-9} K. The Gauss–Newton fit (43) of the objective (42) on the 652 fully economized or fully mechanical samples returns the chiller plant's loss scale within 10^{-6} of 1, the registered unit, and the distribution fraction within 10^{-9} of zero, with the 21 blend samples excluded. The least-squares fit (44) returns the IT heat capacity and the IT-to-air conductance to 3.1×10^{-3} and 1.5×10^{-4} relative, which is the error of reading a BDF2 record at a 900 s step with a forward difference, not an identification failure. Read this as a well-posedness check: the telemetry was produced, noise-free, by the model being identified, so recovery to the discretization error of the records shows that these parameters are identifiable from this kind of record and that the estimators are correct, and says nothing yet about noisy or partially instrumented data. The heat capacity was identifiable only because the record is transient: the IT temperature moves by 13.3 K over the week.



thick gray: cold (7.9 °C outdoors); dashed: mild (18.9 °C outdoors); dotted: hot (27.9 °C outdoors); dot: the best setpoint

Figure 6: Total facility power at steady state against the forced supply setpoint, as the increase over the best setpoint in each weather, at an IT load of 110 kW. The least totals are 118.79 kW (cold), 119.00 kW (mild) and 142.48 kW (hot), all at the 17.9 °C floor. Every curve rises monotonically, more steeply above 19.9 °C, where the server fans begin to ramp; the three curves differ by at most 75 W and lie on top of one another.

Table 2: Survival of the best supply setpoint across the registered chiller-plant choices. Each column is the margin (35) in one weather: the smallest extra power any other setpoint costs over the best one; a negative margin would move the best setpoint. The first row is the registered plant, the next eight change one choice each, and the last is the combination with the smallest margin among all 96.

plant choice changed	cold [W]	mild [W]	hot [W]	verdict
none (the registered plant)	95.8	95.8	86.5	holds
loss scale 0.8	95.8	95.8	88.6	holds
loss scale 1.2	95.8	95.8	84.3	holds
least-power staging	95.8	95.8	86.5	holds
four units of 50 kW	95.8	95.8	75.4	holds
no head-pressure floor	95.8	95.8	75.5	holds
floor 311.75 K less 16.7 K per unit load	95.8	95.8	86.9	holds
fixed condenser floor 296.45 K	95.8	95.8	77.4	holds
constant-flow chilled water	95.8	95.8	88.0	holds
joint worst: loss scale 1.2, four units of 50 kW, no head-pressure floor	95.8	95.8	64.5	holds

Table 3: Parameters recovered from the week's telemetry against the values that generated it. The supply policy is read from all 673 samples, the loss scale and distribution fraction from the 652 fully economized or fully mechanical samples, and the IT heat capacity and conductance from 672 forward differences. Errors at round-off are stated as bounds.

parameter	truth	recovered	error
supply approach a [K]	3	3.000	below 10^{-9} K
supply floor T_{min} [K]	291	291.000	below 10^{-9} K
supply cap T_{max} [K]	300	300.000	below 10^{-9} K
chiller loss scale s	1	1.000000	below 10^{-6}
distribution fraction f_d	0	0.000000	below 10^{-9}
IT heat capacity C_{it} [J/K]	1.0×10^{7}	1.0031×10^{7}	3.1×10^{-3} relative
IT-to-air conductance UA_{ia} [W/K]	1.5×10^{4}	1.5002×10^{4}	1.5×10^{-4} relative

5 Discussion

The first limit is in the supply policy the week runs. The policy follows the outdoor air with a fixed approach up to its cap and takes no feedback from the room, so on the hottest afternoon the supply air sits at its cap of 26.9 °C, the room air reaches 31.7 °C and the IT equipment 39.8 °C. A hall that warm would in practice throttle its compute, raise an alarm or be given a colder setpoint; this model does none of those things, since the compute load follows its own daily cycle whatever the temperatures do and the only response of the hall to its own temperature is the leakage term, 2597 W at that hour. The hottest hours of the week are therefore what this policy produces, and their PUE is the PUE of a hall left to warm. The setpoint sweep, its sensitivity table and the plant fits do not depend on the week's trajectory: the sweep solves steady states at stated constant conditions, and the plant fits read the plant's own inputs and outputs.

The telemetry is synthetic, noise-free and produced by the model that is identified, so the identification establishes well-posedness, not robustness; measurement noise, sensor bias, missing channels, held-out weeks and comparison with other estimators are out of scope. Each mass is a single well-mixed lump, with no plenum, wall or chilled-water loop, and every heat flow is linear in a temperature difference. The plant model has no link from the supply setpoint to the chiller's efficiency, since chilled water enters at a fixed temperature whatever the setpoint; a model in which a warmer supply allowed warmer chilled water could favor a warmer setpoint in hot weather, and the conclusion that the best setpoint is the same in every weather holds for this model only. The chiller law keeps the shape measured on one water-cooled centrifugal chiller and is applied to an air-cooled machine, whose condenser air runs hotter than the condenser water of the test data: 290 of the 350 running mechanical samples (83 %), carrying 89 % of the chiller energy, lie outside the conditions the law was measured over. Head-pressure control, holding condensing at or above 40 °C (a manufacturer default target, not a code value), sets the condenser for 30 % of the chiller energy. Staged unloading and short-cycling losses are not represented, and the chilled water is priced at its 54 °F rating temperature. The survival analysis varies only the registered plant choices; the hall's constants, the load and the supply policy are held.

6 Conclusion

This report asked three questions of a three-mass model of a data-center hall of about 100 kW of IT, cooled by an air-side economizer and two code-minimum air-cooled chillers and driven by a synthetic week of weather and load. What PUE the week produces and why it swings: a mean of 1.121, between 1.017 and 1.268, with the economizer carrying at least part of the cooling in 51 % of the week; the swing is the plant's change from free cooling at an effective COP of 33 to the chiller at down to 3.5, while the hall followed the weather and the supply policy, the room air moving by 10.4 K and the slab by 8.2 K and the IT equipment reaching 39.8 °C on the hottest afternoon. Which forced supply setpoint minimizes total facility power: the cold floor, 17.9 °C, in cold, mild and hot weather alike, a setpoint of 32.9 °C costing 6.24 kW more in cold weather, 6.24 kW in mild and 6.17 kW in hot, because the outdoor temperature, not the setpoint, decides how much of the duty the economizer carries; the answer holds in all 96 registered combinations of chiller-plant choices, with a smallest margin of 65 W, for a plant model in which the setpoint does not change chiller efficiency. Whether the parameters can be recovered from the telemetry: from noise-free records of the same model, the supply policy exactly, the chiller loss scale within 10^{-6} and the distribution fraction within 10^{-9} from 652 samples, and the IT heat capacity and conductance to within 0.31 %, a well-posedness check rather than a calibration. The next steps are to close the loop between the

room's temperature and the supply policy, to repeat the identification on noisy and incomplete records and on held-out weeks, and to couple the supply setpoint to the chilled-water temperature.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level1_building/weather_driven_pue
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
```

`run.py` simulates the week, solves the sweep, scores the plant combinations and fits the parameters, and writes `results.json`, which carries every number and series in this report; `make_report.py` reads only that file. No randomness is drawn (seed 0); the run takes seconds. The gate test is `tests/plugins/datacenter/test_weather_driven_pue_case_study.py`.

B Data and problems

The weather, humidity and load are the analytic series (1)–(3); no external data are used. The chiller law's loss-coefficient ratios and its calibration ranges come from the HVAC plugin's committed extract of a public laboratory test of one water-cooled centrifugal chiller; the minimum efficiencies are those of ASHRAE Standard 90.1-2019, Table 6.8.1-3, Path A, for air-cooled chillers below 150 tons. The study leaves no corpus problems.

C Notation

Table 4 lists the symbols used in this report.

Table 4: Notation.

symbol	meaning	unit
$T_{it}, T_{air}, T_{slab}$	temperatures of the IT equipment, room air and slab	K
$C_{it}, C_{air}, C_{slab}$	heat capacities of the three masses	J/K
$UA_{ia}, UA_{sa}, UA_{env}, G$	conductances IT–air, slab–air, envelope, cooling	W/K
$Q_{ia}, Q_{sa}, Q_{env}, Q_{cool}$	heat flows on those couplings; Q_{cool} is the cooling duty	W
T_{db}, T_{amb}, T_{sup}	outdoor dry bulb; outdoor and supply temperature in the network	K
$P_{base}, P_{fan}, P_{IT}$	IT compute load, server-fan power, IT power	W
α, T_{ref}	leakage coefficient and its reference temperature	W/K, K
$a, T_{min}, T_{max}, T_{\times}, \Delta_b$	supply approach, floor, cap, crossover, blend band	K
m	mechanical share of the duty	–
f_e, f_d	economizer and distribution power per watt of duty	–
P_{cool}, P_{plant}, P_u	cooling power; chiller-plant power; one unit's power	W
n, k, C_u, q_u, x	units installed and running; unit capacity, duty, load fraction	–, –, W, W, –
T_{chi}, T_{ci}	chilled water entering the evaporator; air entering the condenser	K
b_1, b_2, b_3, s	entropy generation, heat leak, exchanger resistance; loss scale	W/K, W, K/W, –
PUE, COP	power usage effectiveness; coefficient of performance	–

References

[Brady et al. 2013] Brady, G. A., Kapur, N., Summers, J. L., Thompson, H. M. (2013). A case study and critical assessment in calculating power usage effectiveness for a data centre. Energy Conversion and Management 76, 155–161. doi:10.1016/j.enconman.2013.07.035.

[Zhang et al. 2014] Zhang, H., Shao, S., Xu, H., Zou, H., Tian, C. (2014). Free cooling of data centers: a review. Renewable and Sustainable Energy Reviews 35, 171–182. doi:10.1016/j.rser.2014.04.017.

[Bacher and Madsen 2011] Bacher, P., Madsen, H. (2011). Identifying suitable models for the heat dynamics of buildings. Energy and Buildings 43(7), 1511–1522. doi:10.1016/j.enbuild.2011.02.005.

[Chua et al. 1996] Chua, H. T., Ng, K. C., Gordon, J. M. (1996). Experimental study of the fundamental properties of reciprocating chillers and their relation to thermodynamic modeling and chiller design. International Journal of Heat and Mass Transfer 39(11), 2195–2204. doi:10.1016/0017-9310(95)00335-5.

[ASHRAE 2019] ASHRAE (2019). ANSI/ASHRAE/IES Standard 90.1-2019: Energy Standard for Buildings Except Low-Rise Residential Buildings. ASHRAE, Atlanta, GA.