Annual savings of an air-side economizer for a data-center hall, priced hour by hour on a staged air-cooled chiller plant

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Abstract

A data center pays for electricity twice, once to run its computers and again to remove their heat, and an air-side economizer removes part of that heat with outdoor air instead of with a chiller. Its value is commonly estimated from an annual-average outdoor temperature, which at a coastal California site lies below the temperature at which the economizer stops carrying the load. This report prices an economizer hour by hour through a year of reanalysis weather and regional demand at a 1000 kW hall in Santa Clara, CA, and asks what it saves, whether the saving pays back the retrofit within a 1.2-year hurdle, and whether the average-based estimate agrees. Each of the 8,751 hours of the record is priced twice, with the economizer and without it, on a plant of two air-cooled chiller units of 1000 kW on the Gordon–Ng law anchored to the ASHRAE 90.1 minimum efficiencies. The economizer carries the whole load in 7,119 hours and part of it in 456 more, and saves 828.7 MWh of the 1,188.3 MWh the chiller-only plant would draw, 69.7 %, worth $124,301 a year; annual PUE falls from 1.2183 to 1.0661. Against a capital cost of $150,000 that is a payback of 1.207 years, which misses the hurdle by 2.4 days, while the average-based estimate predicts 86.1 % savings and a payback of 0.977 years, and clears it. A three-mass transient model of the hall, solved at 96 sampled hours, puts the error of the audit's duty assumption at 1.80 % of the annual cooling electricity. The pair of verdicts breaks under four of eight single changes to the plant model and at the joint worst of 96 combinations. The study is a forward one: every input is stated and nothing is estimated from data.

1 Introduction

Data centers turn nearly all the electricity their computers draw into heat, and they spend further electricity removing it. Masanet, Shehabi, Lei, Smith and Koomey (2020) put global data-center electricity use at about 1 % of world demand and show that efficiency gains, cooling among them, have held that share nearly flat while computing has grown; Barroso, Hölzle and Ranganathan (2019) describe the facility overhead of warehouse-scale computing and the power usage effectiveness (PUE) ratio by which it is reported. Cooling is the largest part of that overhead in most halls, and the cheapest way to reduce it is to stop refrigerating when the outdoor air is already cold enough. An air-side economizer does exactly that: it brings in outdoor air, filters and mixes it, and holds the supply temperature with fans alone while the weather allows, engaging the chiller only when it does not. Whether that is worth its capital cost at a given site is a question about a year of weather, not about a design point.

The energy that economizers save has been estimated for many climates. Siriwardana, Jayasekara and Halgamuge (2013) price air-side economizers for Australian cities from hourly weather and report savings that vary strongly between sites; Ham, Kim, Choi and Jeong (2015) compare air-side

economizer types in a modular data center and find the annual saving to depend on the control that decides when the economizer is used. Both work hour by hour, because the hours a site spends above the economizer's limit are not distributed evenly and are not the hours at which the hall's load is average. What the economizer does not cover is priced by the chiller, and a chiller's efficiency depends on its load and on the temperature it rejects heat to. The standard thermodynamic description of that dependence is due to Gordon and Ng, whose model Chua, Ng and Gordon (1996) tested against measurements on reciprocating chillers and Foliaco, Bula and Coombes (2020) revisited on a field-measured centrifugal machine; it ties compressor power to the load, to the two water temperatures and to three loss coefficients. The efficiency level of a new machine is bounded from below by the minimum full-load and integrated part-load efficiencies of ASHRAE Standard 90.1 (ASHRAE, 2019), rated at the conditions of AHRI Standard 550/590 (AHRI, 2023).

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses its data-center and chiller-plant plugins to write the supply policy, the economizer and the staged chiller of one hall, to price a year of real weather through them hour by hour, and to solve the hall's three-mass thermal network as a check on the hourly audit. Section 3 states the framework and each of these constructs precisely.

This report makes no claim of novelty. It asks what an air-side economizer saves at one coastal California hall over one year of weather, whether that saving clears a 1.2-year payback hurdle, and whether the annual-average estimate that such a decision is usually made with gives the same answer. It answers by pricing every hour of the record twice, with the economizer and without it, on a staged plant of code-minimum air-cooled chillers, and it adds two checks: one on the assumption that the hall's cooling duty is its computing load, and one on how far the answer depends on the choices made in modeling the plant. It is a forward study: every input is stated and nothing is estimated from data. Section 2 states the hall, the data and the model, Section 3 the methods, Section 4 the results, Section 5 their limits and Section 6 the answers; the appendices hold the reproduction recipe and the provenance of the inputs.

2 System and data

The hall. Figure 1 draws the hall and its plant. It is a single data-center hall of 1000 kW of computing capacity at Santa Clara, CA (37.37°N, 121.92°W). Its cooling is delivered as supply air whose setpoint floats with the weather: the supply temperature follows the outdoor dry bulb by an approach of 3 K, bounded below at 291 K (17.9°C) and above at 297 K (23.9°C), the upper bound being the limit the equipment is operated to. The outdoor temperature at which the approach reaches that upper bound, 20.85°C, is the crossover: below it outdoor air alone holds the setpoint and the economizer carries the whole load for the cost of its fans and pumps, 3 % of the heat removed; above it the chiller takes a share that grows linearly and reaches the whole load a blend band of 2 K higher. The counterfactual is the same hall and the same plant with the economizer removed, so that the chiller serves every hour.

The plant. The chiller plant is two identical air-cooled units of 1000 kW (284 tons of refrigeration) each. Each unit is the least efficient machine that meets both ASHRAE 90.1 minimums for its class

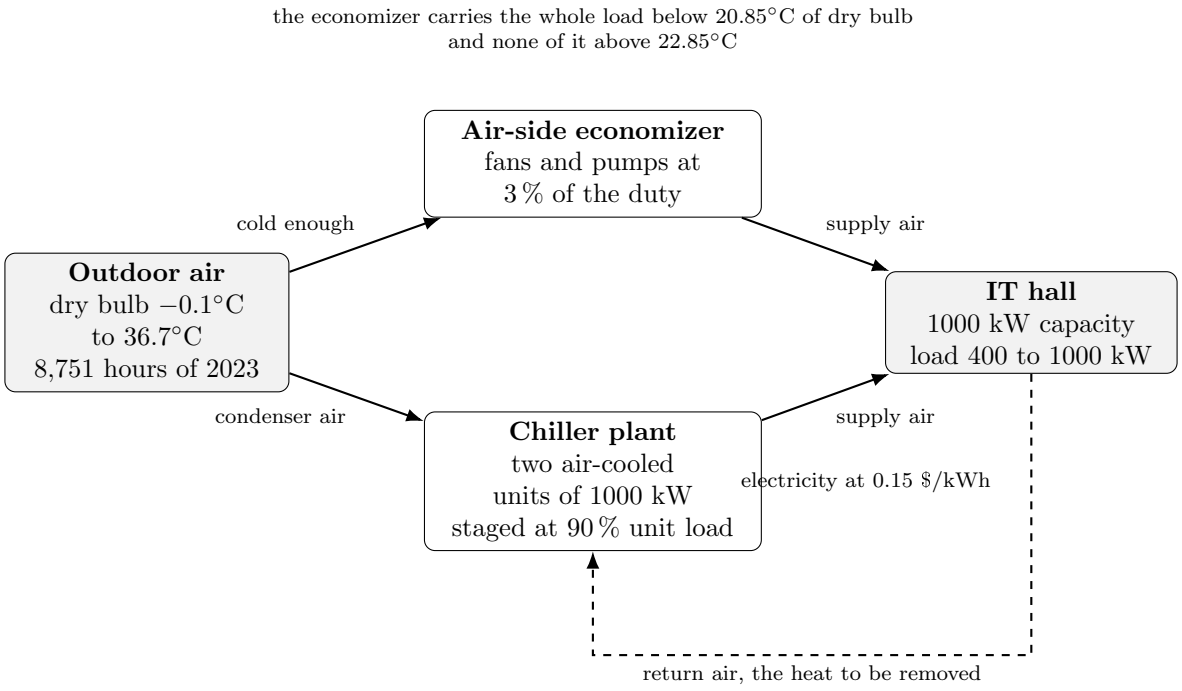


Figure 1: The hall of the study and the two ways its heat is removed. Outdoor air is cold enough to hold the supply temperature without refrigeration below 20.85°C of dry bulb, and the economizer then carries the whole load for the cost of its fans and pumps; above 22.85°C the chiller plant carries all of it, and between the two the load is shared. The counterfactual the report prices against is the same hall and the same plant with no economizer, the chiller serving every hour. The hall's computing load follows the regional demand shape and its weather is a reanalysis of the site's year.

and size, its full-load and its integrated part-load efficiency, which for this size are COP 2.96 (10.1 EER) and COP 4.10 (14.0 EER); the part-load minimum is the one that binds. An air-cooled unit rejects its heat to the outdoor dry bulb itself, with no cooling tower and no approach, and its fans hold the condensing temperature at or above a floor whatever the weather. A second unit is staged on when the load passes 90 % of the running unit's capacity. Section 3 states the law, the anchoring and the floor.

The data. Two public hourly records of calendar year 2023, joined on their timestamps, give 8,751 usable hours of the 8,760 in the year; the rest are gaps in the join and are dropped. The weather is the ERA5 reanalysis (Hersbach et al., 2020) at the site, read from the Open-Meteo historical archive: 2 m dry-bulb temperature, which runs from −0.1°C to 36.7°C with an annual mean of 15.26°C, and 2 m relative humidity, from which the wet bulb is computed and runs from −2.3°C to 22.0°C. A reanalysis is a physical model of the atmosphere fitted to the observations of the period, so its hourly temperatures are consistent with each other and with the season, and they are not the readings of a station at the site. The computing load is not metered at any real hall; it is built from the hourly system demand of the California grid operator, which runs from 12,537 to 43,869 MW over the year, normalized onto a utilization band from 40 % to full capacity and multiplied by the hall's capacity. The resulting load runs from 400 to 1,000 kW with a mean of 622 kW, and it correlates with the dry bulb at 0.498: the hall is assumed to compute hardest when the region does, which is when it is hot. That correlation is the reason the question cannot

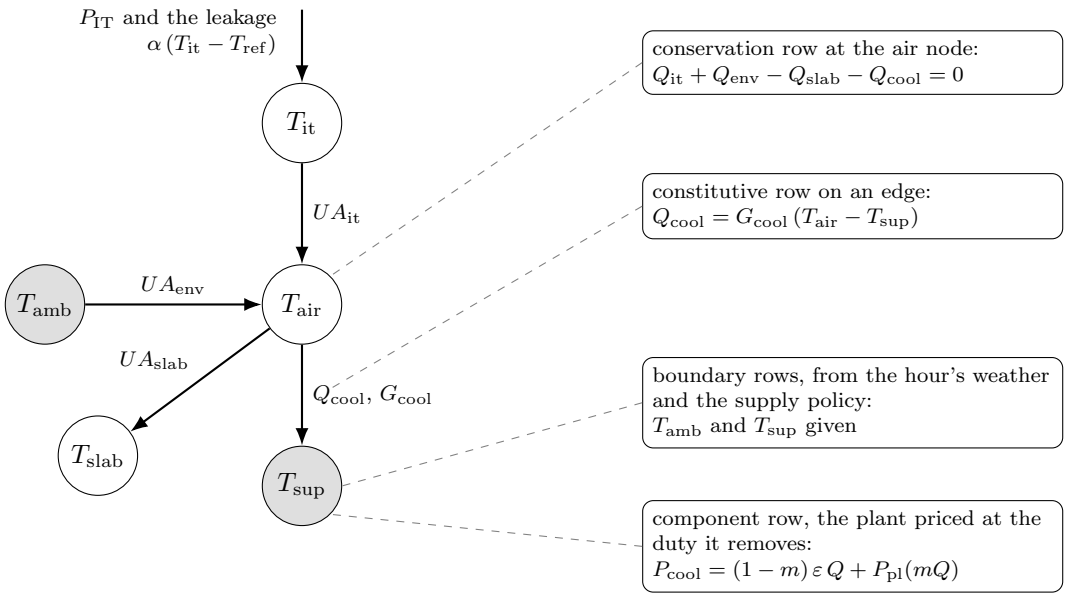


Figure 2: The model of the hall. White circles are unknown temperatures of the three heat-storing masses — the room air, the computing equipment and the slab — and gray circles are boundaries fixed by the hour's weather and by the supply policy; arrows are heat flows along conductance edges. The rows of the model are of four kinds: conservation at every node, a constitutive law on every edge, the boundary values, and the component law that prices the plant at the heat it removes. The hourly audit of this report uses the component law with the duty set to the computing power; the three-mass network, solved with $C_{\mathrm{air}} = 16$ MJ/K, $C_{\mathrm{it}} = 80$ MJ/K and $C_{\mathrm{slab}} = 2400$ MJ/K, is what checks that assumption.

be answered at an average. No number in this report is fitted to these data; they are inputs.

The model. Figure 2 shows the model of the hall. The hourly audit treats the hall as quasi-static: at each hour the heat to be removed is the computing load of that hour, and the plant is priced at that duty and that weather. The check on that assumption is the three-mass thermal network of the same figure, in which the room air, the computing equipment and the slab each store heat, exchange it through conductances, gain heat from the equipment and from the outdoor air across the envelope, and lose it to the supply air. Solved to steady state it returns a cooling duty that differs from the computing load by the equipment's temperature-dependent leakage and by the heat crossing the envelope.

Assumptions.

- Units: temperatures in K in the relations and in degrees Celsius in the text, powers in W, energies in MWh, money in United States dollars at a flat 0.15 \$/kWh with no demand charge and no time-of-use structure.
- The hour is the resolution: every quantity is held constant within it, and the hall is in equilibrium at each hour, so no heat is carried from one hour to the next.
- The hall's cooling duty is its computing load; Section 4 bounds the error of that assumption.

- The economizer is limited by dry-bulb temperature alone: no humidity limit, no outdoor-air quality limit and no minimum outdoor-air fraction is modeled, and its fans and pumps cost a fixed fraction of the heat they move.
- The retrofit costs \$150 per kW of computing capacity, \$150,000 in all, spent once; no maintenance, downtime, escalation or discounting enters the payback.
- The plant's units are identical, share their load equally, and are priced by a law calibrated on one test machine and scaled by similarity to this size.

The question. What does the economizer save over the year, in electricity and in money, and does that saving return the \$150,000 within the 1.2-year hurdle the facility applies? Does the annual-average estimate give the same answer? And how far do those answers depend on the choices made in modeling the chiller plant, and on the climate the hall will see?

3 Methods

3.1 The hourly audit

The supply policy, the economizer and the chiller are three relations evaluated at every hour t of the record. The supply setpoint floats with the outdoor dry bulb T_{db} by the approach α_{a} between the two bounds (2); the crossover (1) is the outdoor temperature at which the upper bound is reached, and above it the chiller's share m of the duty rises linearly over the blend band Δ (3). The heat to be removed is the computing load (4), and the plant's electrical power is the economizer's parasitic fraction ε on the share it carries plus the chiller plant's power at the mechanical share of the duty and at the outdoor temperature it rejects heat to (5),

$$T_{\times} = T_{\max} - \alpha_{\mathrm{a}}, \tag{1}$$

$$T_{\mathrm{sup}}(t) = \min\big(T_{\max},\ \max(T_{\min},\ T_{\mathrm{db}}(t) + \alpha_{\mathrm{a}})\big), \tag{2}$$

$$m(t) = \mathrm{clip}\big((T_{\mathrm{db}}(t) - T_{\times})/\Delta,\ 0,\ 1\big), \tag{3}$$

$$Q(t) = P_{\mathrm{IT}}(t), \tag{4}$$

$$P_{\mathrm{cool}}(t) = \big(1 - m(t)\big)\,\varepsilon\,Q(t) + P_{\mathrm{pl}}\big(m(t)\,Q(t),\ T_{\mathrm{db}}(t)\big). \tag{5}$$

with $\alpha_{\mathrm{a}} = 3$ K, $T_{\min} = 291$ K, $T_{\max} = 297$ K, $\Delta = 2$ K and $\varepsilon = 0.03$. The counterfactual is the same relations with the crossover forced below every temperature of the record, so that $m = 1$ at every hour and the chiller serves the whole year; nothing else changes. Summing the hourly power over the record gives each case's annual cooling electricity (6), their difference the saving (7) and its fraction of the chiller-only case (8), and the annual PUE of a case is its total electricity over its computing electricity (9). The simple payback of the retrofit is its capital cost K over the saving valued at the electricity rate c (10), and the decision is whether it is at or below the hurdle H,

$$E_{\mathrm{cool}} = \textstyle\sum_t P_{\mathrm{cool}}(t)\,\Delta t, \tag{6}$$

$$S = E^{\mathrm{ch}}_{\mathrm{cool}} - E^{\mathrm{ec}}_{\mathrm{cool}}, \tag{7}$$

$$\phi = S/E^{\mathrm{ch}}_{\mathrm{cool}}, \tag{8}$$

$$\mathrm{PUE} = (E_{\mathrm{IT}} + E_{\mathrm{cool}})/E_{\mathrm{IT}}, \tag{9}$$

$$\Pi = K/(c\,S). \tag{10}$$

with $\Delta t = 1$ h, $K = \$150,000$, $c = 0.15$ \$/kWh and $H = 1.2$ yr. The superscripts mark the two cases, economizer and chiller only.

3.2 The network model

The relations of the previous subsection and the thermal network of the hall below are two instances of one construction, in which the unknowns of a system and the relations they must satisfy are written on a graph and solved together; this subsection states it. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{11}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{12}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in this report is estimated from data: every input is stated, and the results are the solutions of these relations and quantities computed from them. Everything that follows is a choice of unknowns and relations on this graph.

3.3 The chiller plant

A chiller removing heat Q from chilled water that enters its evaporator at T_{chi}, rejecting it to a medium entering its condenser at T_{ci}, draws compressor power $P = Q/\mathrm{COP}$. The Gordon–Ng law, derived from the first and second laws applied to the refrigeration cycle and tested against measurements by Chua et al. (1996), relates the three through three loss coefficients: b_1 (W/K), the internal entropy generation, b_2 (W), an equivalent heat leak, and b_3 (K/W), the total heat-exchanger resistance. Solved for the power, the law (13) gives (14), and below a minimum stable load $Q_{\min}$, here 10 % of the unit's capacity, the machine cycles on and off and its time-averaged power is the linear (15),

$$\frac{T_{\mathrm{chi}}}{T_{\mathrm{ci}}}\Big(1 + \frac{1}{\mathrm{COP}}\Big) = 1 + \frac{b_1 T_{\mathrm{chi}}}{Q} + \frac{b_2\,(T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{ci}}\,Q} + \frac{b_3\,Q}{T_{\mathrm{ci}}}\Big(1 + \frac{1}{\mathrm{COP}}\Big), \tag{13}$$

$$P_{\mathrm{GN}}(Q) = \frac{T_{\mathrm{ci}}\,Q + b_1 T_{\mathrm{chi}} T_{\mathrm{ci}} + b_2\,(T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{chi}} - b_3\,Q} - Q, \tag{14}$$

$$P_{\mathrm{unit}}(Q) = \frac{Q}{Q_{\min}}\,P_{\mathrm{GN}}(Q_{\min}) \quad \text{for } Q < Q_{\min}. \tag{15}$$

With all three coefficients zero the law is the Carnot limit. The heat-exchanger term makes the losses grow with load, so the COP rises from low load toward a maximum and falls again beyond it.

The shape of the law, the ratios between the three coefficients, is taken from a fit to the test chiller of ASHRAE research project RP-1043, a machine of capacity C^{RP}, whose fitted coefficients are

$$b_1^{\mathrm{RP}} = 78.43 \text{ W/K}, \tag{16}$$

$$b_2^{\mathrm{RP}} = 121,971 \text{ W}, \tag{17}$$
$$b_3^{\mathrm{RP}} = 7.711 \times 10^{-5} \text{ K/W}, \tag{18}$$
$$C^{\mathrm{RP}} = 90 \text{ tons}. \tag{19}$$

The fit (16)–(19) is carried to a unit of capacity C by similarity, the entropy generation and the heat leak growing with capacity and the exchanger resistance falling with it, and one factor s multiplies all three to set the efficiency level,

$$b_1 = s\, b_1^{\mathrm{RP}}\, C/C^{\mathrm{RP}}, \tag{20}$$
$$b_2 = s\, b_2^{\mathrm{RP}}\, C/C^{\mathrm{RP}}, \tag{21}$$
$$b_3 = s\, b_3^{\mathrm{RP}}\, C^{\mathrm{RP}}/C, \tag{22}$$

The factor is anchored to the code. ASHRAE 90.1 requires an air-cooled chiller to meet both a full-load and an integrated part-load minimum, rated at the AHRI 550/590 air-cooled conditions: chilled water leaving at 44°F and entering at 54°F at full load, with air entering the condenser at 95/80/65/55°F at the 100/75/50/25 % load points, weighted 0.01/0.42/0.45/0.12 into the integrated value. Each minimum alone fixes the factor at which the unit just meets it, and the least efficient unit meeting both takes the smaller. For the 1000 kW unit that is $s = 0.6942$, set by the part-load minimum: its integrated value equals the minimum, 4.103, while its full-load COP at the rating point, 4.405, exceeds the full-load minimum of 2.960. Through (20)–(22) that gives the coefficients the plant of this report is priced with,

$$b_1 = 172.01 \text{ W/K}, \tag{23}$$
$$b_2 = 267,516 \text{ W}, \tag{24}$$
$$b_3 = 1.694 \times 10^{-5} \text{ K/W}. \tag{25}$$

In operation the unit's chilled water is held at its rating temperature, 285.37 K entering, and the condenser sees the outdoor dry bulb, floored by head-pressure control (26). The floor is the manufacturer's fan-control target, a saturated condensing temperature of 40°C expressed on entering air through a design condenser approach of $\beta = 16.7$ K that scales with the running unit's load fraction x, so that $T_{\mathrm{f}} = 313.15$ K at full load. The plant stages by threshold (27) and shares the duty equally among the running units (28), with $n = 2$ units and $\sigma = 0.90$,

$$T_{\mathrm{ci}} = \max\big(T_{\mathrm{db}},\ T_{\mathrm{f}} - \beta\, x\big), \tag{26}$$
$$n_{\mathrm{run}} = \min\big(n,\ \lceil mQ/(\sigma C)\rceil\big), \tag{27}$$
$$P_{\mathrm{pl}} = n_{\mathrm{run}}\, P_{\mathrm{unit}}\big(mQ/n_{\mathrm{run}}\big). \tag{28}$$

The floor is what keeps an air-cooled machine from being priced at an impossible efficiency on a cold night, and it is applied both in operation and at the rating points where the unit is anchored, because the machine is rated with its own controls active.

3.4 The thermal network of the hall

The audit's duty (4) is an assumption: it takes every watt the computers draw to leave as heat through the cooling, and nothing else to enter or leave. The hall's thermal network tests it. Three masses store heat — the room air, the computing equipment and the slab — and their balances, with the conductance law of the cooling coil, are

$$C_{\mathrm{it}}\, \dot{T}_{\mathrm{it}} = P_{\mathrm{IT}} + \alpha\,(T_{\mathrm{it}} - T_{\mathrm{ref}}) - UA_{\mathrm{it}}\,(T_{\mathrm{it}} - T_{\mathrm{air}}), \tag{29}$$

$$C_{\mathrm{air}}\,\dot{T}_{\mathrm{air}} = UA_{\mathrm{it}}\,(T_{\mathrm{it}} - T_{\mathrm{air}}) + UA_{\mathrm{env}}\,(T_{\mathrm{amb}} - T_{\mathrm{air}}) - UA_{\mathrm{slab}}\,(T_{\mathrm{air}} - T_{\mathrm{slab}}) - Q_{\mathrm{cool}}, \tag{30}$$

$$C_{\mathrm{slab}}\,\dot{T}_{\mathrm{slab}} = UA_{\mathrm{slab}}\,(T_{\mathrm{air}} - T_{\mathrm{slab}}), \tag{31}$$

$$Q_{\mathrm{cool}} = G_{\mathrm{cool}}\,(T_{\mathrm{air}} - T_{\mathrm{sup}}). \tag{32}$$

where $\alpha\,(T_{\mathrm{it}} - T_{\mathrm{ref}})$ is the leakage power of the silicon, which rises with its temperature and is negative below the reference. At steady state the slab carries no net heat and sits at the air temperature, and the cooling duty is the computing power plus the leakage plus whatever crosses the envelope, so it exceeds (4) in hot weather and falls short of it in cold. The constants are $C_{\mathrm{air}} = 16$ MJ/K, $C_{\mathrm{it}} = 80$ MJ/K, $C_{\mathrm{slab}} = 2400$ MJ/K, $UA_{\mathrm{it}} = 120$ kW/K, $UA_{\mathrm{slab}} = 40$ kW/K, $UA_{\mathrm{env}} = 4$ kW/K, $G_{\mathrm{cool}} = 200$ kW/K, $\alpha = 1600$ W/K and $T_{\mathrm{ref}} = 300$ K. The network is solved to steady state at the four hours nearest the 10th, 50th, 90th and 99th percentiles of dry bulb, each at that hour's load, supply temperature and outdoor temperature, and again at 96 hours spaced evenly through the record; in the latter the cooling electricity of the audit's duty and of the solved duty are summed and compared, which is the aggregate that matters for the headline.

3.5 The average-based estimate

The estimate this report compares against is the one a business case is usually built on: a single representative hour, at the annual-average outdoor condition and the annual-average computing load. The same relations (2)–(5) are evaluated once at that hour in both cases, their ratio taken as the saving fraction, and that fraction applied to the chiller-only annual energy, which is the same number the hourly audit produces,

$$\bar{T} = \langle T_{\mathrm{db}} \rangle, \tag{33}$$

$$\bar{Q} = \langle P_{\mathrm{IT}} \rangle, \tag{34}$$

$$\hat{\phi} = 1 - P^{\mathrm{ec}}_{\mathrm{cool}}(\bar{Q}, \bar{T}) / P^{\mathrm{ch}}_{\mathrm{cool}}(\bar{Q}, \bar{T}), \tag{35}$$

$$\hat{S} = \hat{\phi}\; E^{\mathrm{ch}}_{\mathrm{cool}}. \tag{36}$$

so that the two estimates differ only in the fraction, not in the base they are applied to. The payback follows from $\hat{S}$ through (10). Nothing about this estimate is ill-posed: it is the correct saving for a year whose every hour is the average hour.

3.6 Sensitivity to the plant model

The verdict rests on a plant whose every choice a reader could make differently, so the report scores the verdict again under each of them. This is a sensitivity analysis over modeling choices, not a statement about how uncertain the answer is: each combination is a different model, priced exactly, and the table says which models keep the verdict and which overturn it. Five choices are varied: loss scale, staging rule, unit count, heat-sink floor, chilled-water model. The loss scale multiplies the anchored unit's three coefficients by 0.8, 1, 1.2. The staging rule is the threshold rule (27) or least power, which runs the number of units that draws least among those that can carry the load. The unit count is n units of the registered capacity or $2n$ of half of it, each re-anchored to the code minimums for its own size. The condenser floor is the registered fan-control target, or no condenser floor, a condenser floor set by a compressor pressure ratio of 2.8, a condenser floor fixed at 296.45 K whatever the load. The chilled-water model holds the entering water at its rating temperature or lets it fall with load under constant evaporator flow. Their product is 96 combinations. In each, the plant is rebuilt and the whole year is re-priced in both cases, and the verdict is scored by two

clauses, one per case, each written so that it is positive while that case keeps the verdict it has under the registered plant,

$$c_{\mathrm{h}} = \operatorname{sign}(\Pi_{\mathrm{h}}^{0} - H)\,(\Pi_{\mathrm{h}} - H), \tag{37}$$

$$c_{\mathrm{a}} = \operatorname{sign}(\Pi_{\mathrm{a}}^{0} - H)\,(\Pi_{\mathrm{a}} - H), \tag{38}$$

$$g = \min(c_{\mathrm{h}},\ c_{\mathrm{a}}). \tag{39}$$

with Π_{h} the payback of the hourly audit, Π_{a} that of the average-based estimate and the superscript 0 marking the registered plant. The margin g is positive while both verdicts hold and negative once either has turned. The table reports the registered plant, every combination that differs from it in one choice, and the joint worst, the combination with the smallest margin; ties would go to the combination with the lowest level indices in the order the choices are listed here.

3.7 Sensitivity to the climate

The same year is re-priced with every hour's dry bulb raised by 3, 6 K, the relative humidity of each hour held as recorded and the wet bulb recomputed, and the computing load left unchanged. This is a sensitivity to a uniform shift, not a climate projection: it says how the two estimates move as the site warms, and in particular what happens once the annual mean itself crosses the crossover (1).

3.8 Metrics

The savings question is answered by the annual cooling electricity of each case (6) in MWh and by their difference (7), valued at the electricity rate: that is what the meter records and what the retrofit is bought for, and its reference is zero, no saving. It is reported beside the saving fraction (8), whose reference is the 86.1 % the average-based estimate predicts, and beside the annual PUE (9), dimensionless, whose reference is the 1.2183 of the chiller-only case. The investment question is answered by the simple payback (10) in years, read against the hurdle $H = 1.2$ yr: at or below it the retrofit is approved, above it it is not. The audit's duty assumption is measured by the relative difference between the cooling electricity computed from the assumed duty and from the solved duty, in percent, read against zero and judged by whether it is small next to the gap between the two estimates it is meant to support. The plant's operating range is measured by the share of chiller electricity priced at conditions outside the range the law was calibrated over, in percent, whose reference is zero and whose value here is a limitation to be stated rather than a result. The robustness of the pair of verdicts is measured by the margin g of (39) in years, read against zero: its sign says whether both verdicts survive a combination of plant choices, and its size says by how much.

4 Results

4.1 The year at the site

Figure 3 plots the dry-bulb temperature of the record against the percent of hours at or below it, with the crossover, the top of the blend band and the annual mean drawn as horizontal lines and the share of hours above the crossover as a vertical one. The annual mean, 15.26°C, lies 5.59 K below the crossover, and the curve crosses the crossover at the 81.4th percentile: 1,632 hours of the year (18.6 %) are warmer than the economizer can fully serve, and 1,176 of them are warmer than

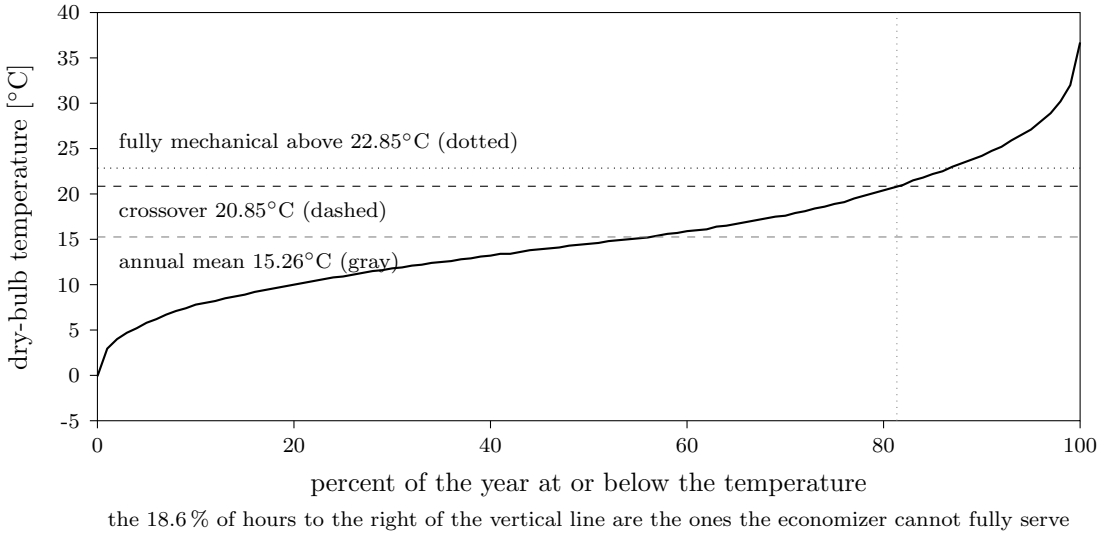


Figure 3: Dry-bulb temperature of the year against the percent of hours at or below it, one point per percentile. The dashed horizontal line is the crossover, the outdoor temperature above which the economizer can no longer hold the supply setpoint on its own; the dotted horizontal line is the temperature above which the chiller carries the whole load; the gray line is the annual mean, and the vertical line marks the share of hours above the crossover. The annual mean lies 5.59 K below the crossover, so an estimate made at the mean sees a year of free cooling, while 1,632 hours of the year are above it.

it can serve at all. The distribution is the reason the two estimates of this report differ: a mean well inside the free-cooling region coexists with a fifth of the year outside it. Table 1 and Figure 4 break the year into bins of 5 K of dry bulb. Panel (a) of the figure stacks, for each bin, the hours the economizer carries and the hours of mechanical cooling; panel (b) plots the mean computing load of the hours in the bin. The mechanical hours appear only at or above 20°C, and the mean computing load of the bins rises monotonically from there, from 591 kW in the bin holding most of the year to 820 kW in the warmest. Over the whole record the load and the dry bulb correlate at 0.498. The hours the economizer cannot serve are therefore both few and heavy: they average 702 kW against 604 kW in the hours it can, at a mean dry bulb of 25.4°C against 12.9°C.

4.2 What the economizer saves

Table 2 gives the year in both cases and Figure 5 shows it month by month: panel (a) the cooling electricity of the chiller-only case and of the economizer case in each month, panel (b) the electricity each month saves against that month's mean dry bulb. With the economizer the hall spends 359.6 MWh on cooling against 1,188.3 MWh without it, a saving of 828.7 MWh, 69.7 % of the chiller-only cost and $124,301 a year at 0.15 $/kWh; annual PUE falls from 1.2183 to 1.0661. The economizer carries the whole load in 7,119 hours of the 8,751 and part of it in 456 more, and the chiller carries the whole load in 1,176. In the chiller-only case the monthly cost is nearly flat, because it follows the computing load and not the weather; in the economizer case it rises from 11.8 to 64.9 MWh between the coldest and the warmest month, and panel (b) shows the saving falling as the month's mean temperature rises. The saving is not distributed as the hours are. In the 7,119 hours the economizer fully carries, it saves 815.2 MWh, 98.4 % of the total; in the 1,632

Table 1: The year by dry-bulb bin. Mechanical hours are the sum of each hour's chiller share, so they total 1,394 of the year's 8,751 hours; the last two columns are the cooling electricity of the two cases in the bin. Every mechanical hour lies at or above 20°C, and the mean computing load of those bins is the highest of the year.

dry bulb [°C]	hours	mechanical hours	mean load [kW]	economizer [MWh]	chiller only [MWh]
−5 to 0	1	0	593	0.0	0.1
0 to 5	304	0	597	5.4	40.1
5 to 10	1,436	0	592	25.5	188.8
10 to 15	2,981	0	591	52.8	391.7
15 to 20	2,154	0	625	40.4	290.0
20 to 25	1,124	643	658	114.5	156.6
25 to 30	563	563	732	86.5	86.5
30 to 35	163	163	808	29.6	29.6
35 to 40	25	25	820	4.9	4.9

Table 2: The year in both cases. Computing energy is 5,443.0 MWh in both, so the two differ only in the cost of removing that heat. The economizer saves 828.7 MWh, 69.7 % of the chiller-only case, worth $124,301 a year at 0.15 $/kWh, and lowers annual PUE by 0.1522. Hours are counted by the share of each hour the chiller carries: an hour is fully economized when that share is zero.

quantity	with the economizer	chiller only
cooling electricity [MWh]	359.6	1,188.3
of it the chiller [MWh]	226.0	1,188.3
of it the economizer [MWh]	133.6	0.0
cooling cost [$/yr]	$53,941	$178,242
annual PUE	1.0661	1.2183
hours fully economized	7,119	0
hours sharing the load	456	0
hours fully mechanical	1,176	8,751

hours it does not, it saves 13.4 MWh. Those hours hold 64.1 % of the economizer case's whole cooling bill while being 18.6 % of the year. The reason is the correlation of the previous subsection together with the plant: the mechanical hours carry the year's heaviest computing load, and the chiller that serves them rejects heat to air at 25.4°C on average, where it runs at a COP of 4.38 against the 4.40 of its rating point.

4.3 The average-based estimate and the payback

At the annual-average condition — 15.26°C dry bulb, 11.20°C wet bulb, 622 kW of computing load — the plant is in free cooling: the single representative hour draws 18.7 kW with the economizer and 134.2 kW without it, a saving fraction of 86.1 % (35). Applied to the same chiller-only annual energy that is the base of both estimates, that is 1,023.1 MWh and $153,461 a year, against the 828.7 MWh and $124,301 the hourly audit gives. The average-based estimate overstates the saving by $29,160 a year, 23.5 % of the hourly figure, and it does so for a definite reason: the average hour is not a representative hour of a quantity that is nonlinear in temperature and correlated with load. It reports 100 % free cooling for a year in which 18.6 % of the hours have some mechanical cooling. The two estimates fall on opposite sides of the decision. Against the $150,000 of capital cost the average-based saving pays back in 0.977 years and clears the 1.2-year hurdle by 81.3 days;

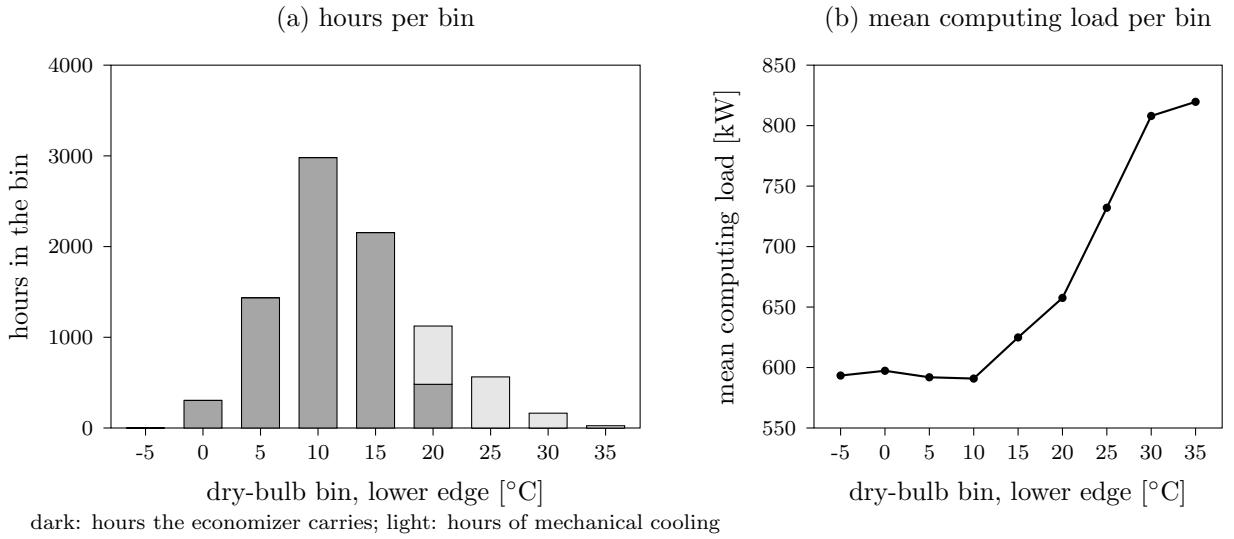


Figure 4: The year in nine bins of 5 K of dry bulb. (a) Hours in each bin, split into the hours the economizer carries (dark) and the hours with mechanical cooling (light); the split is not a bar per hour but the sum of each hour's mechanical share. (b) The mean computing load of the hours in each bin. The mechanical hours sit entirely in the warm bins, and those are the bins in which the hall computes hardest: the mean load rises from 593 kW in the coldest bin to 820 kW in the warmest.

the hourly saving pays back in 1.207 years and misses it by 2.4 days. The verdict of this report is therefore that the retrofit does not clear the hurdle as stated, and that the estimate which says it does is wrong by more than the width of the decision. The second half of that sentence is the robust part: the gap between the estimates is $29,160 a year, while the hourly payback misses the hurdle by 0.0067 yr, a margin of 0.6 % of the hurdle itself. A verdict that rests on such a margin is fragile, and the rest of this section measures how fragile.

4.4 The duty assumption

Table 3 compares the audit's duty, the computing load of the hour, with the duty the three-mass network solves for at the same load, supply temperature and outdoor temperature, at the hours nearest the 10th, 50th, 90th and 99th percentiles of dry bulb. The solved duty is lower than the assumed one in cold weather, by 9.49 % at the coldest hour of the four, because the equipment sits below its reference temperature and the envelope loses heat to the outdoors; it is higher in hot weather, by 3.49 % at the hottest, where both terms reverse. The errors are of opposite sign, and they largely cancel in a sum over the year: over 96 hours sampled evenly through the record the cooling electricity computed from the two duties is 4.06 and 3.98 MWh, a difference of 1.80 %. That is small next to the 23.5 % gap between the two estimates it is meant to support, and it is not small next to the 0.6 % by which the payback misses the hurdle.

4.5 The range the plant is priced over

The chiller law is calibrated on the benchmark steady states of one test machine, and the box those points span runs from 288.6 to 303.2 K on the condenser side, from 0.24 to 1.18 of rated load and from 4.9 to 20.8 K of lift. The year puts the chiller outside that box in 842 of the 1,632 hours it

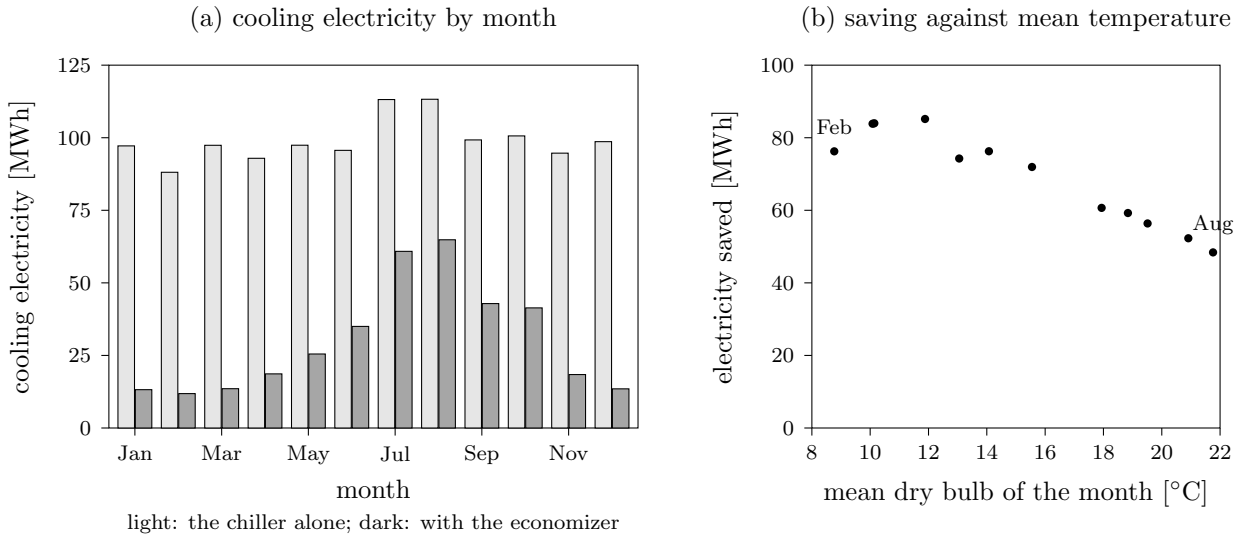


Figure 5: Cooling electricity month by month. (a) The chiller-only case (light bars) and the economizer case (dark bars) in each month of the year. (b) The electricity each month saves, against the month's mean dry bulb, one point per month, with the coldest and the warmest month named. The chiller-only case is nearly flat across the year because its load follows the computing load, while the economizer case rises fourfold from Feb to Aug, and the saving falls as the month warms.

runs, carrying 49.3 % of the chiller's electricity, almost always because the condenser air is warmer than the calibrated condenser water: the site reaches 36.7°C and the box ends at 30.1°C. A further 74.0 % of the chiller's electricity is drawn in hours where the head-pressure floor (26) is what sets the condenser temperature rather than the weather, and 5.9 % in hours below the calibrated load range. These shares do not make the pricing wrong, but they are extrapolations of a fit, and they are the largest single reason to treat the absolute savings of this report as indicative of a site rather than as a quotation for a machine.

4.6 Sensitivity to the plant model

Table 4 lists the registered plant, each single change of a plant choice and the joint worst of the 96 combinations, with both paybacks, the margin (39) and whether the pair of verdicts survives. Under the registered plant the margin is 0.0067 yr, which is the hourly payback's distance above the hurdle; the average-based clause is far larger, 0.2226 yr, so the margin is set entirely by how narrowly the hourly payback misses. Four of the eight single changes break the pair: chilled water falling with load takes the hourly payback to 1.133 yr and the average-based payback to 0.918 yr; no condenser floor takes the hourly payback to 1.687 yr and the average-based payback to 1.236 yr; twice as many units of half the capacity takes the hourly payback to 1.166 yr and the average-based payback to 0.958 yr; the unit's losses scaled by 1.2 takes the hourly payback to 1.028 yr and the average-based payback to 0.832 yr. Four leave it standing, with margins from 0.0088 to 0.0799 yr. At the joint worst — the unit's losses scaled by 0.8, least-power staging, no condenser floor — the margin is −0.4226 yr and both paybacks lengthen, to 2.249 and 1.623 years, so the pair breaks from the other side: the retrofit fails the hurdle on both estimates. The lesson is not symmetric between the two findings. The verdict that the hourly payback misses the hurdle holds in every row of the table but the three in which a less lossy or more finely staged plant lifts the saving enough to clear

Table 3: The hourly audit's duty assumption against the three-mass network, at the hours nearest the 10th, 50th, 90th and 99th percentiles of dry bulb. The audit takes the duty to be the computing load; the network solves it from the balances of the three masses, so it adds the temperature-dependent leakage of the equipment and the heat crossing the envelope, which are negative in cold weather and positive in hot. Over 96 hours sampled evenly through the year the two duties give cooling electricity of 4.06 and 3.98 MWh, a difference of 1.80 %.

hour	dry bulb [°C]	computing load [kW]	duty of the audit [kW]	duty of the solve [kW]	difference [%]
cold (P10)	7.8	561.0	561.0	507.7	9.49
median (P50)	14.5	631.0	631.0	604.3	4.22
warm (P90)	24.2	600.2	600.2	597.8	0.41
hot (P99)	32.0	823.8	823.8	852.5	3.49

Table 4: The pair of payback verdicts across the plant choices: the registered plant, each single change, and the joint worst of all 96 combinations. Each row rebuilds the plant, re-prices the whole year in both cases and recomputes both paybacks against the 1.2-year hurdle; the margin is the smaller of the two clauses of (39) and is positive while the hourly audit still misses the hurdle and the average-based estimate still clears it. Four of the eight single changes break that pair, and so does the joint worst.

plant choice changed	hourly [yr]	average-based [yr]	margin g [yr]	verdict
none (the registered plant)	1.207	0.977	+0.0067	holds
losses $\times 0.8$	1.457	1.180	+0.0195	holds
chilled water falls with load	1.133	0.918	−0.0671	flips
no condenser floor	1.687	1.236	−0.0362	flips
a condenser floor set by a compressor pressure ratio of 2.8	1.221	0.987	+0.0208	holds
a condenser floor fixed at 296.45 K whatever the load	1.280	1.023	+0.0799	holds
twice the units, half the capacity	1.166	0.958	−0.0345	flips
least-power staging	1.209	0.983	+0.0088	holds
losses $\times 1.2$	1.028	0.832	−0.1723	flips
losses $\times 0.8$, least-power staging, no condenser floor	2.249	1.623	−0.4226	flips

it, while the finding that the average-based estimate overstates the saving holds in every row of it.

4.7 Sensitivity to the climate

Table 5 re-prices the same year with every hour's dry bulb raised uniformly. The hourly saving falls from 69.7 to 60.7 and 46.2 % of the chiller-only cost as the site warms by 3 and 6 K, and the share of hours with any free cooling falls from 86.6 to 65.6 %. The average-based estimate does not track it. At +3 K the average hour is still below the crossover, so the estimate returns the same 86.1 % as before and now overstates the saving by 41.8 %; at +6 K the average hour itself crosses the crossover, the representative hour is priced as a mechanical one, and the estimate collapses to 21.4 %, understating the saving by 53.6 %. The error of an average is not a bias of fixed sign: it is whatever the shape of the year happens to make it, and it changes sign as a threshold moves across the mean.

Table 5: The same year with every hour's dry bulb raised by a fixed amount, each hour's relative humidity held as recorded and its wet bulb recomputed. The hourly saving falls from 69.7 to 46.2 % of the chiller-only cost as the site warms. The average-based estimate does not follow it: it overstates the saving by 41.8 % at +3 K and then understates it by 53.6 % at +6 K, where the warmed annual mean has itself crossed the crossover and the single average hour is priced as a mechanical one.

warming [K]	hours with free cooling [%]	hourly saving [%]	average-based saving [%]	difference [%]
+0	86.6	69.7	86.1	+23.5
+3	78.2	60.7	86.1	+41.8
+6	65.6	46.2	21.4	−53.6

5 Discussion

Two findings of this report have different standing, and it is worth separating them. That an annual-average business case overstates an economizer's saving at this site is robust: it follows from a mean that sits 5.59 K inside the free-cooling region while 18.6 % of the hours lie outside it, from the correlation of load with temperature, and from a chiller whose efficiency falls exactly where it is needed; it survives every plant model in Table 4 and it grows under warming before it changes sign. That the retrofit fails a 1.2-year hurdle is not robust: the hourly payback misses the hurdle by 2.4 days, 0.6 % of the hurdle, which is inside the 1.80 % the duty assumption itself is worth, and four of eight single changes of plant model turn it. A facility reading this report should take the $29,160 correction seriously and the pass-or-fail verdict as a statement that the retrofit sits on the hurdle, not below it.

The model stops in several places. The computing load is not metered: it is the regional demand shape scaled onto a utilization band, which reproduces the diurnal and seasonal pattern of a load that follows the region but not the idiosyncrasies of a real hall, and both the correlation with temperature and the load in the hot hours would change with a different workload. The economizer is limited by dry bulb alone: no humidity bound, no air-quality bound and no minimum outdoor-air fraction is represented, and a real economizer at this site would lose some of the hours this report grants it, which moves the answer against the retrofit. The hall is quasi-static, so thermal mass neither delays nor shifts the load; the three-mass check bounds the cost of that at 1.80 % of the annual cooling electricity in the sampled aggregate, and it does not examine what pre-cooling the mass could buy. The chiller law prices 49.3 % of its electricity outside the range it was calibrated over, mostly because a California afternoon is hotter than the condenser water of the test machine, and a further 74.0 % in hours where the head-pressure floor and not the weather sets the condenser temperature; the floor is a manufacturer default, and its choice is one of the changes in Table 4 that turns the verdict.

The economics are deliberately plain. Electricity is a flat 0.15 $/kWh, so the hot afternoon hours the economizer cannot serve are valued no higher than a winter night, although those are the hours a California tariff prices highest and the hours a demand charge is set in; a time-of-use tariff would widen the gap between the two estimates, because the hours the average hides are the expensive ones. The capital cost is a single number per kilowatt of computing capacity and carries no contingency, and the payback is undiscounted. Nothing in the report speaks to reliability, to the space the economizer needs, or to what it does to the hall during a wildfire.

6 Conclusion

This report asked what an air-side economizer saves at a 1000 kW data-center hall on the California coast over a year of real weather and real demand shape, whether the saving returns its capital cost within a 1.2-year hurdle, and whether the annual-average estimate such decisions are usually made with agrees. Priced hour by hour through 8,751 hours on a plant of two code-minimum air-cooled chillers, the economizer carries the whole load in 7,119 hours and saves 828.7 MWh of the 1,188.3 MWh the chiller alone would draw, 69.7 %, worth $124,301 a year, and it lowers annual PUE from 1.2183 to 1.0661. Against $150,000 of capital cost that is a payback of 1.207 years, which misses the 1.2-year hurdle by 2.4 days, so the retrofit does not clear the hurdle as stated — but it sits on it, and four of eight single changes to the plant model move it to the other side. The average-based estimate does not agree: it sees the annual mean 5.59 K below the crossover, predicts 100 % free cooling and 86.1 % savings, and overstates the annual saving by $29,160, 23.5 %, because the hours it cannot see are the hot, heavily loaded hours in which the chiller is least efficient. That overstatement survives every plant model examined and grows to 41.8 % under 3 K of uniform warming, then reverses sign once the warmed mean itself crosses the crossover. The hall's three-mass thermal network puts the cost of the audit's assumption that the cooling duty is the computing load at 1.80 % of the annual cooling electricity. The next step is to price the same hours against a time-of-use tariff and a demand charge, which value the hours the average hides at more than the flat rate used here, and to give the economizer the humidity and air-quality limits a real one is operated under.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level1_building/real_weather_free_cooling
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
cd ../../../../..
python -m pytest -q tests/plugins/datacenter/test_real_weather_free_cooling_case_study.py
```

`run.py` reads the committed hourly snapshot, prices every hour in both cases, runs the four network solves and the 96-hour aggregate check, re-prices the year under the 96 plant combinations and under the climate shifts, and writes `results.json`; `make_report.py` reads only `results.json` and writes `report.tex`. No random numbers are drawn and no value is fitted. The test in the last line regenerates the results, compares them with the committed file and rebuilds the tex.

B Data and problems

The hourly snapshot `data/santa_clara_2023_hourly.csv` holds 8,751 rows from 2023-01-01 08:00:00+00:00 to 2023-12-31 23:00:00+00:00, with columns for the timestamp, dry-bulb temperature, relative humidity, wind speed and regional demand. The weather columns are the ERA5 reanalysis read from the Open-Meteo historical archive at 37.37°N, 121.92°W, licensed CC BY 4.0; the demand column is the California system operator's published hourly demand for the same year, resampled from its five-minute record. The wet bulb is computed from dry bulb and relative humidity, not read. The chiller law's coefficients are the fit to the ASHRAE RP-1043 test chiller carried in the plant model, and the efficiency minimums are those of ASHRAE 90.1-2019, Table 6.8.1-3, Path A, for air-cooled chillers. The study leaves one open problem: the hours in Table 4 that turn the payback verdict differ from one another mainly in how the condenser is floored, and no measurement of a real air-cooled machine at this site has been brought to bear on that choice.

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