Ranking containment retrofits on a data-center campus with a shared chiller plant by site energy and by PUE

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Abstract

A data-center campus pays for electricity twice, once to run its computers and again to remove their heat, and operators commonly rank efficiency investments by power usage effectiveness (PUE), the ratio of site power to computing power. This report asks which of three buildings' airflow-containment retrofits saves the most electricity at the campus meter when the buildings share one chilled-water plant, and whether a lowest-PUE rule agrees. The buildings hold 30 racks each and differ only in how much hot exhaust air recirculates to the rack inlets; a retrofit reduces one building's recirculation. Each scenario is solved as a steady heat-balance network of racks, aisles, rooms and buildings sized so that each link carries its design heat at a stated temperature difference, pinned at the 61°F chilled water the plant supplies, with rack power rising with rack temperature, and the heat is priced with a staged plant of two chiller units on the Gordon–Ng law, anchored to the ASHRAE 90.1 minimum efficiencies. At baseline the campus draws 37.68 kW of computing power and 6.46 kW of chiller power at a chiller COP of 5.83, a PUE of 1.17140. Retrofitting the worst-contained building saves 1.8 MWh/yr, against 1.4 and 0.5 MWh/yr for the other two, and is the recommendation. Every retrofit raises campus PUE (to 1.17146, 1.17160, 1.17166), because the chiller's COP falls as its load falls, so a lowest-PUE rule picks the smallest saver. The site saves 1.12 W per W saved at the racks for the recommended retrofit, which the constant-COP estimate $1 + 1/\mathrm{COP} = 1.17$ overstates. The lowest-PUE pick holds under each of six single changes to the plant model and in all 48 combinations, by margins no larger than the retrofits' own PUE changes. Racks run between 309.7 and 313.8 K in every scenario. The study is a forward one: every input is stated and nothing is estimated from data, and each scenario is one linear solve of 339 unknowns.

1 Introduction

Data centers turn nearly all the electricity their computers draw into heat, and they spend further electricity removing it; Barroso, Hölzle and Ranganathan (2019) discuss the energy efficiency of warehouse-scale facilities, including this cooling overhead. The overhead is reported as power usage effectiveness, PUE, the site's total electrical power divided by the power that reaches the computing equipment, so that 1.0 would mean no overhead at all. PUE is simple to measure and to compare, and it is widely used to judge investments in cooling. It is also a ratio, and a ratio can move the wrong way when its denominator changes: Brady, Kapur, Summers and Thompson (2013) examine how PUE is calculated for a working data center and caution against reading it as a measure of overall energy efficiency. The question matters most where a decision changes the computing power itself, and airflow containment is such a decision.

Hot exhaust air that recirculates to the inlets of the racks raises the temperature of the equipment, and the power a server draws rises with its temperature through fan speed and semiconductor

leakage. Sharma, Bash, Patel, Friedrich and Chase (2005) show that where heat is generated in a data center and how the cooling responds to it change the energy the facility spends, and they propose managing workload with the thermal state in view. Containment, the separation of cold supply aisles from hot exhaust aisles, attacks recirculation directly. Its effect at the campus meter then passes through the chilled-water plant, whose efficiency depends on its load. Chiller part-load efficiency has a thermodynamic description due to Gordon and Ng, whose model Chua, Ng and Gordon (1996) test against measurements on reciprocating chillers; it ties compressor power to the load, the two water temperatures, and three loss coefficients. The efficiency level of new chillers is bounded from below by the minimum full-load and integrated part-load efficiencies of ASHRAE Standard 90.1 (ASHRAE, 2019), rated at the conditions of AHRI Standard 550/590.

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses its data-center plugin to write the heat balances of the racks, aisles, rooms and buildings of a three-building campus on one chilled-water boundary, and its chiller-plant model to price the heat the plant removes. Section 3 states the framework and each of these constructs precisely.

This report makes no claim of novelty. It asks one question of a campus of three buildings that share a chiller plant: which building's containment retrofit saves the most electricity at the campus meter, and whether ranking the retrofits by the lowest resulting PUE gives the same answer. It answers by solving the campus's steady heat balance once per scenario and pricing the heat with a staged plant of code-minimum chillers, and it adds a check of how far the PUE ranking depends on the choices made in modeling the plant. It is a forward study: every input is stated and nothing is estimated from data. Section 2 states the campus, the data and the model, Section 3 the methods, Section 4 the results, Section 5 their limits and Section 6 the answers; the appendices hold the reproduction recipe and the provenance of the inputs.

2 System and data

The campus. Figure 1 draws the campus. Each of its three buildings has three rooms, each room one row of ten racks, so 30 racks per building and 90 in all. Every rack draws a base power of 400 W plus 1.5 W for every kelvin its temperature exceeds 300 K. A row has a cold aisle, the air the racks take in, and a hot aisle, the air they exhaust; the row's mixing factor is the conductance of the path from hot aisle to cold aisle divided by that of the supply path from the room, zero for perfect containment. The baseline mixing factors are 0.4 in building A, 1.5 in building B, 4 in building C; a containment retrofit sets one building's factor to 0.2 and leaves the others as they are. All buildings reject their heat to one chilled-water loop held at 289.26 K (61°F), the water that enters the plant's units, whatever the load.

The plant. The chilled water is produced by two identical water-cooled positive-displacement chiller units of 30 kW (8.53 tons of refrigeration) each, 60 kW in all. Each unit is the least efficient machine that meets both ASHRAE 90.1 minimums for its class and size, its full-load and its integrated part-load efficiency. The plant's condenser water enters at 85°F (302.6 K), the AHRI full-load rating condition, in every scenario, and a second unit is staged on when the load passes 90 % of the running unit's capacity.

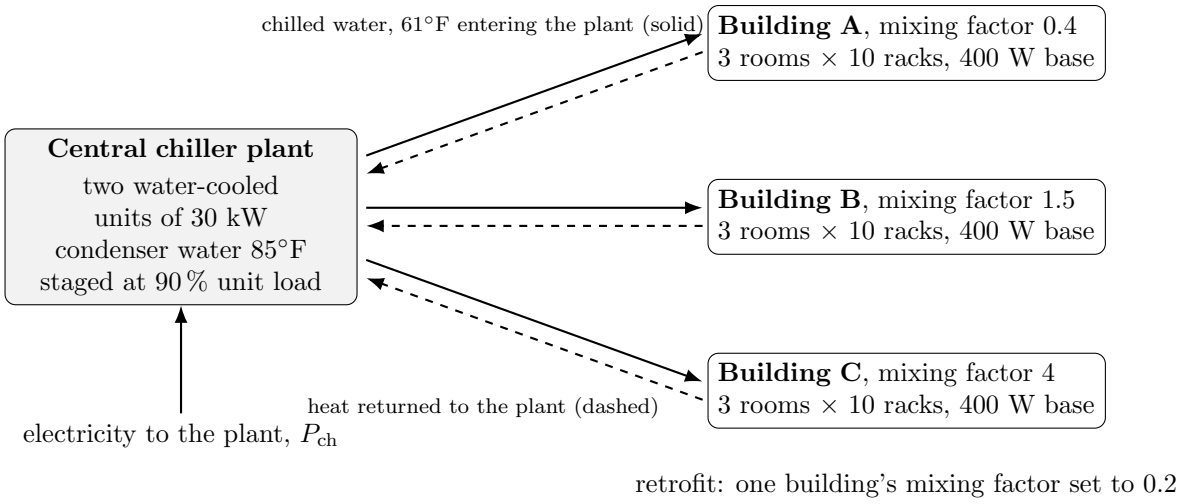


Figure 1: The campus of the study. One central plant of two identical water-cooled chiller units serves three buildings through a shared chilled-water loop held at the plant's design entering water, 289.26 K (solid arrows); the heat of the racks returns to the plant (dashed arrows) and the plant draws electricity to reject it. The buildings are identical except for the mixing factor of their rows, the ratio of recirculated hot air to supplied cold air: A is well contained, B fair, C poor. A retrofit sets one building's mixing factor to 0.2.

The data. No measured data enter the study. It is a comparison of specified scenarios: every input is a stated parameter, every output follows from the model, and nothing is drawn at random, so the results are exactly reproducible. The chiller's part-load shape is the one fitted to the published test data of a 90-ton centrifugal chiller (ASHRAE research project RP-1043), and its efficiency level is set by the code minimums; both are described in Section 3.

The model. Figure 2 shows the model of one row with its room and building. The unknowns are the temperatures of every rack, aisle, room and building, the power of every rack, and the heat flow on every edge; with 90 racks the network has 339 of them and as many rows. The plant is not part of the network: the heat the buildings reject is priced by the plant law after the network is solved, which is exact here because the chilled-water temperature is held at the plant's design entering water and so does not depend on the plant's load.

Assumptions.

- Units: temperatures in K, powers and heat flows in W, conductances in W/K; site figures are quoted in kW and annual energy in MWh/yr for 8,760 hours of operation at the solved point.
- Steady state: every stored quantity is constant, so the heat rejected to the chilled water equals the computing power.
- Rooms and rows within a building are identical, the racks in a row are identical, and the workload is the same in every scenario.
- The chilled-water boundary is held at the plant's design entering temperature at every load; the plant does not feed back on it.

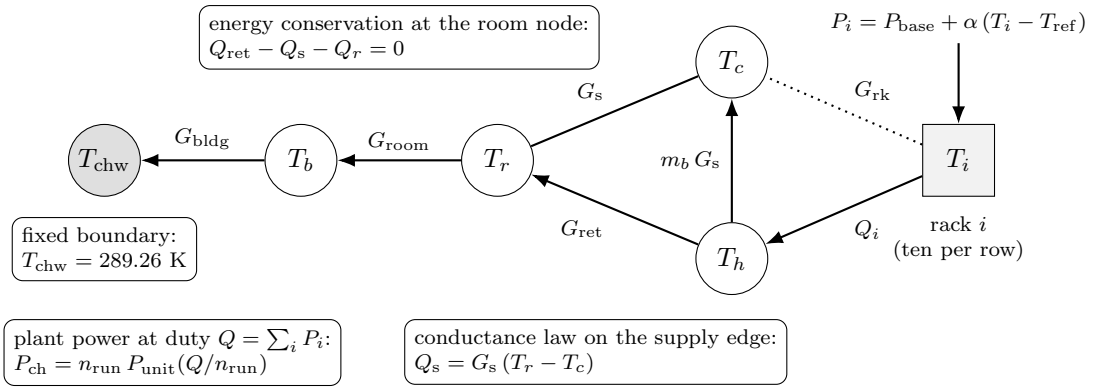


Figure 2: The model of one row, room and building. Circles are unknown temperatures, the shaded circle the fixed chilled-water boundary, the square a rack; arrows are heat flows on conductance edges (the dotted edge sets the rack's heat flow from its inlet air, the cold aisle, and delivers it to the hot aisle). The rows of the model are of four kinds: energy conservation at every node, a conductance law on every edge, the fixed chilled-water boundary, and the plant power priced at the heat the buildings reject. The rack's power rises with its temperature.

- The plant's condenser water is fixed at the rating condition; there is no weather.
- The retrofit changes only the mixing factor; its cost, construction and downtime are outside the model.

The question. Which retrofit saves the most electricity at the campus meter, and does the retrofit with the lowest resulting campus PUE coincide with it? A supporting question is how robust the lowest-PUE pick is to the choices made in modeling the plant.

3 Methods

3.1 The campus thermal network

The unknowns of one building b are, for every rack i, its temperature T_i, electrical power P_i and heat flow Q_i; for every row, the cold-aisle and hot-aisle temperatures T_c and T_h and the supply, mixing and return heat flows Q_{s}, Q_{m} and Q_{ret}; for every room, its temperature T_r and the heat flow Q_r it passes to the building; and the building's temperature T_b and the heat flow Q_b it rejects to the chilled water at T_{chw}. Each physical law is a residual that vanishes when the law holds. Energy conservation at the rack (1), the rack's power law (2), the conductance law of its heat path (3), conservation at the two aisles (4) and (5), the conductance laws of the supply, mixing and return paths (6)–(8), and conservation and conductance at the room (9), (10) and the building (11), (12) give

$$r_i^{\text{rack}} = P_i - Q_i, \tag{1}$$

$$r_i^{\text{load}} = P_i - P_{\text{base}} - \alpha\,(T_i - T_{\text{ref}}), \tag{2}$$

$$r_i^{\text{cool}} = Q_i - G_{\text{rk}}\,(T_i - T_c), \tag{3}$$

$$r^{\text{cold}} = Q_{\text{s}} + Q_{\text{m}}, \tag{4}$$

$$r^{\text{hot}} = \textstyle\sum_i Q_i - Q_{\text{m}} - Q_{\text{ret}}, \tag{5}$$

$$r^{\mathrm{sup}} = Q_{\mathrm{s}} - G_{\mathrm{s}}\,(T_r - T_c), \tag{6}$$
$$r^{\mathrm{mix}} = Q_{\mathrm{m}} - m_b\,G_{\mathrm{s}}\,(T_h - T_c), \tag{7}$$
$$r^{\mathrm{ret}} = Q_{\mathrm{ret}} - G_{\mathrm{ret}}\,(T_h - T_r), \tag{8}$$
$$r^{\mathrm{room}} = Q_{\mathrm{ret}} - Q_{\mathrm{s}} - Q_r, \tag{9}$$
$$r^{\mathrm{rl}} = Q_r - G_{\mathrm{room}}\,(T_r - T_b), \tag{10}$$
$$r^{\mathrm{bldg}} = \textstyle\sum_r Q_r - Q_b, \tag{11}$$
$$r^{\mathrm{bl}} = Q_b - G_{\mathrm{bldg}}\,(T_b - T_{\mathrm{chw}}), \tag{12}$$

where the sums run over the racks of the row and the rooms of the building. The rack's heat flow is driven by its temperature above the cold aisle it draws air from and is delivered to the hot aisle. The supply flow Q_{s} is counted from room to cold aisle and is negative when the aisle is the warmer; the mixing path carries heat from the hot aisle back to the cold aisle with a conductance m_b times the supply's, which is how recirculation raises the air the racks take in. The constants are $P_{\mathrm{base}} = 400$ W, $\alpha = 1.5$ W/K, $T_{\mathrm{ref}} = 300$ K, $G_{\mathrm{rk}} = 33.33$ W/K, $G_{\mathrm{s}} = 333.33$ W/K, $G_{\mathrm{ret}} = 333.33$ W/K, $G_{\mathrm{room}} = 1,333.33$ W/K, $G_{\mathrm{bldg}} = 4,000$ W/K and $T_{\mathrm{chw}} = 289.26$ K, with the mixing factors m_b of Section 2. The conductances follow one sizing rule: each link carries the design heat of its racks, 400 W per rack, at a stated temperature difference, a 12 K rise across the rack's air stream (G_{rk} for one rack, G_{s} and G_{ret} for the row's 10) and 3 K approaches at the room and building coils (G_{room}, G_{bldg}). For the whole campus the rows number 339, as many as the unknowns, and every one of them is linear, so the system has one solution, which Newton's method reaches from a flat start in a single linear solve up to round-off.

Because the racks, rows and rooms of a building are identical, the solution has a closed form that shows how containment acts. Eliminating the aisle flows from (4)–(8) puts the cold aisle $NP_b m_b/((1+m_b)G_{\mathrm{ret}} + m_b G_{\mathrm{s}})$ above the room, with $N = 10$ racks per row and $R = 3$ rooms per building; adding the rises across the room, the building and the rack gives the rack's temperature above the chilled water per watt of rack power, k_b, and (2) then fixes the rack power,

$$k_b = \frac{RN}{G_{\mathrm{bldg}}} + \frac{N}{G_{\mathrm{room}}} + \frac{N m_b}{(1+m_b)\,G_{\mathrm{ret}} + m_b\,G_{\mathrm{s}}} + \frac{1}{G_{\mathrm{rk}}}, \tag{13}$$
$$P_b = \frac{P_{\mathrm{base}} + \alpha\,(T_{\mathrm{chw}} - T_{\mathrm{ref}})}{1 - \alpha\,k_b}, \tag{14}$$
$$T_i = T_{\mathrm{chw}} + k_b\,P_b, \tag{15}$$

The mixing factor enters only the third term of (13), and the denominator of (14) amplifies any rise: $k_{\mathrm{A}} = 0.0517$ K/W, $k_{\mathrm{B}} = 0.0562$ K/W, $k_{\mathrm{C}} = 0.0583$ K/W and $k = 0.0493$ K/W after a retrofit give rack powers of 416.1, 419.3, 420.7 and 414.5 W. The solved network reproduces (14) to below 10^{-9} W in every rack of every scenario.

3.2 The network model

The residuals of the previous subsection are one instance of a general construction, in which the unknowns of a network and the relations they must satisfy are written on its graph and solved together; this subsection states it. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With

as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{16}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{17}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in this report is estimated from data: every input is stated, and the results are the solutions of these relations and quantities computed from them. Everything that follows is a choice of unknowns and relations on this graph.

3.3 The chiller plant model

A chiller unit removing heat Q from chilled water that enters its evaporator at T_{chi}, with condenser water entering at T_{ci}, draws compressor power $P = Q/\mathrm{COP}$. The Gordon–Ng law, derived from the first and second laws applied to the refrigeration cycle and tested on reciprocating chillers by Chua et al. (1996), relates them through three loss coefficients: b_1 (W/K), the internal entropy generation, b_2 (W), an equivalent heat leak, and b_3 (K/W), the total heat-exchanger resistance. Solved for the power, the law (18) gives (19),

$$\frac{T_{\mathrm{chi}}}{T_{\mathrm{ci}}}\Big(1 + \frac{1}{\mathrm{COP}}\Big) = 1 + \frac{b_1 T_{\mathrm{chi}}}{Q} + \frac{b_2\,(T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{ci}}\,Q} + \frac{b_3\,Q}{T_{\mathrm{ci}}}\Big(1 + \frac{1}{\mathrm{COP}}\Big), \tag{18}$$

$$P_{\mathrm{GN}}(Q) = \frac{T_{\mathrm{ci}}\,Q + b_1 T_{\mathrm{chi}} T_{\mathrm{ci}} + b_2\,(T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{chi}} - b_3\,Q} - Q, \tag{19}$$

$$P_{\mathrm{cyc}}(Q) = \frac{Q}{Q_{\min}}\,P_{\mathrm{GN}}(Q_{\min}), \tag{20}$$

The heat-exchanger term b_3 makes the losses grow with load, so COP rises from low load toward a maximum; with all three coefficients zero, (18) is the Carnot limit. Below a minimum stable load $Q_{\min} = 3$ kW, a tenth of the unit's capacity, a unit cycles on and off, and its time-averaged power is the linear (20); above it the unit draws $P_{\mathrm{unit}} = P_{\mathrm{GN}}$ from (19).

The shape of the law, the ratios between the three coefficients, is taken from the fit to the test chiller of ASHRAE research project RP-1043, of capacity C^{RP},

$$b_1^{\mathrm{RP}} = 78.43 \text{ W/K}, \tag{21}$$

$$b_2^{\mathrm{RP}} = 121971 \text{ W}, \tag{22}$$

$$b_3^{\mathrm{RP}} = 7.711 \times 10^{-5} \text{ K/W}, \tag{23}$$

$$C^{\mathrm{RP}} = 317 \text{ kW}. \tag{24}$$

It is carried to a unit of capacity C by similarity, entropy generation and heat leak growing with capacity and the exchanger resistance falling with it, and one factor s multiplies all three to set the efficiency level,

$$b_1 = s\,b_1^{\mathrm{RP}}\,C/C^{\mathrm{RP}}, \tag{25}$$

$$b_2 = s\, b_2^{\mathrm{RP}}\, C/C^{\mathrm{RP}}, \tag{26}$$

$$b_3 = s\, b_3^{\mathrm{RP}}\, C^{\mathrm{RP}}/C, \tag{27}$$

The factor is anchored to the code. ASHRAE 90.1 (ASHRAE, 2019) requires a water-cooled positive-displacement chiller below 75 tons to meet both a full-load and an integrated part-load value (IPLV) minimum, here COP 4.69 and COP 5.86. Both are rated at the AHRI 550/590 conditions: chilled water leaving at 44°F with a 10°F rise at full load, and condenser water entering at 85/75/65/65°F at 100/75/50/25 % load. The IPLV (28) is the weighted mean of the COPs at those four points, with a 55°F floor on the condenser water applied as the unit's own head-pressure control. Each minimum alone fixes the factor at which the unit just meets it, s_{FL} or s_{IPLV}, found by bisection because COP falls monotonically as s grows, and the least efficient unit meeting both takes the smaller (29),

$$\mathrm{IPLV} = 0.01\,\mathrm{COP}_{100} + 0.42\,\mathrm{COP}_{75} + 0.45\,\mathrm{COP}_{50} + 0.12\,\mathrm{COP}_{25}, \tag{28}$$

$$s = \min(s_{\mathrm{FL}}, s_{\mathrm{IPLV}}), \tag{29}$$

For the 30 kW unit, $s = 0.5974$: its IPLV equals the minimum, 5.86, while its full-load COP at the rating point, 5.577, exceeds the full-load minimum, so the part-load requirement binds. The resulting coefficients are $b_1 = 4.441$ W/K, $b_2 = 6907$ W and $b_3 = 4.860 \times 10^{-4}$ K/W.

The plant of $n = 2$ units shares its duty equally among the running units and stages them by threshold, (30)–(31), so two units run whenever the load exceeds 0.45 of the plant's 60 kW. In operation the chilled water enters each unit at the plant's design temperature, $T_{\mathrm{chi}} = 289.26$ K (61°F), the temperature at which the network rejects its heat, and leaves at 51°F; this water is 7°F warmer than at the rating point, and on it a unit's full-load COP is 6.251. The condenser water enters at $T_{\mathrm{ci}} = 302.59$ K. In steady state every watt the racks draw is rejected to the chilled water, so the duty is the computing power (32), and the plant's COP (33) and the campus PUE (34) follow. Substituting the one into the other gives (35): in this campus PUE moves only through the plant's COP, PUE rising exactly when COP falls.

$$n_{\mathrm{run}} = \min\left(n, \lceil Q/(0.90\,C)\rceil\right), \tag{30}$$

$$P_{\mathrm{ch}} = n_{\mathrm{run}}\, P_{\mathrm{unit}}(Q/n_{\mathrm{run}}), \tag{31}$$

$$Q = P_{\mathrm{IT}}, \tag{32}$$

$$\mathrm{COP} = Q/P_{\mathrm{ch}}, \tag{33}$$

$$\mathrm{PUE} = (P_{\mathrm{IT}} + P_{\mathrm{ch}})/P_{\mathrm{IT}}, \tag{34}$$

$$\mathrm{PUE} = 1 + 1/\mathrm{COP}, \tag{35}$$

The load band in which the plant is efficient is the range of load fractions at which its COP, evaluated on a grid of the plant's range, is at least 90 % of its best; here it runs from 0.283 of rated capacity up to the rating.

3.4 Scenarios

Four scenarios share every input except the mixing factors: the baseline, and a retrofit of each building in turn that sets its mixing factor to 0.2. Each scenario's network (1)–(12) is solved, its computing power $P_{\mathrm{IT}} = \sum_i P_i$ summed over all 90 racks, and the plant priced at that duty. A retrofit j is compared with the baseline through its site power saving and its computing power saving,

$$\Delta P_{\mathrm{site},j} = (P_{\mathrm{IT}} + P_{\mathrm{ch}})_0 - (P_{\mathrm{IT}} + P_{\mathrm{ch}})_j, \tag{36}$$

$$\Delta P_{\mathrm{IT},j} = P_{\mathrm{IT},0} - P_{\mathrm{IT},j}, \tag{37}$$

$$\Delta E_j = \Delta P_{\mathrm{site},j}\, H/1000, \tag{38}$$

$$\rho_j = \Delta P_{\mathrm{site},j}/\Delta P_{\mathrm{IT},j}, \tag{39}$$

$$\rho_{\mathrm{const}} = 1 + 1/\mathrm{COP}_0, \tag{40}$$

with subscript 0 the baseline, $H = 8,760$ hours of operation a year and ΔE_j in MWh/yr for powers in kW. The ratio ρ_j is the site power saved per unit of rack power saved. If the chiller's COP did not depend on its load, each watt removed at the racks would also remove $1/\mathrm{COP}$ watts at the chiller and the ratio would be ρ_{const}; with a load-dependent COP, differentiating $P_{\mathrm{IT}}(1 + 1/\mathrm{COP})$ gives

$$\frac{dP_{\mathrm{site}}}{dP_{\mathrm{IT}}} = 1 + \frac{1}{\mathrm{COP}} - \frac{P_{\mathrm{IT}}}{\mathrm{COP}^2}\,\frac{d\,\mathrm{COP}}{dP_{\mathrm{IT}}}, \tag{41}$$

which falls below ρ_{const} wherever COP rises with load. The recommendation is the retrofit with the largest ΔE_j; the lowest-PUE rule picks the retrofit with the smallest PUE_j.

3.5 Survival of the lowest-PUE pick

The PUE ranking depends on the slope of the plant's COP with load, and that slope rests on modeling choices a reader could make differently. Five are varied. The loss scale multiplies the anchored unit's three coefficients by 0.8, 1, 1.2. The staging rule is the threshold rule (30) or least power, which runs the number of units that draws least among those that can carry the load. The unit count is n or $2n$ units of half the capacity, each re-anchored to the code minimums for its size. The condenser floor is 55°F or 65°F, applied both at the rating points where the unit is anchored and in operation. The chilled-water model holds the entering water at its design temperature, 61°F, or lets it fall with load under constant evaporator flow, entering at the 51°F leaving temperature plus the design rise times the unit's load fraction. The primary levels are those of the registered plant, and the product of the levels gives 48 combinations. In each, the plant is rebuilt, the four scenarios' solved computing powers are held fixed and re-priced, and the lowest-PUE pick p of the registered plant is scored by

$$c_j = \mathrm{PUE}_j - \mathrm{PUE}_p, \tag{42}$$

$$g = \min_{j \neq p} c_j, \tag{43}$$

over the other two retrofits j. The margin g is positive while p keeps the lowest PUE and negative when another retrofit takes it. The table reports the registered plant, each combination that differs from it in one choice, and the joint worst combination, the one with the smallest margin; ties would go to the combination with the lowest level indices in the order the choices are listed here.

3.6 Metrics

The energy question is answered by the annual site energy saved, ΔE_j from (38), in MWh/yr: it is what the campus meter records and what the retrofit is bought for, so it is the measure a retrofit should be ranked by, and its reference is zero, no saving. The ranking question is answered by the campus PUE of each scenario, (34), dimensionless, read against the baseline PUE; a PUE rule is a sound proxy only if the retrofit with the lowest PUE is also the one with the largest ΔE_j. The coupling through the plant is measured by the ratio ρ_j of (39), watts saved at the site per watt saved at the racks, read against the constant-COP estimate ρ_{const} of (40); by (41) the ratio

Table 1: The four scenarios. PUE is site power over computing (IT) power; the PUE change and the savings are against the baseline; the annual saving assumes the operating point holds for 8,760 hours a year. Every retrofit raises PUE and lowers the chiller's COP while saving electricity.

scenario	PUE	PUE change [10^{-5}]	chiller COP	IT saved [kW]	site saved [kW]	saved [MWh/yr]
baseline	1.17140	—	5.834	—	—	—
retrofit A	1.17146	+6.76	5.832	0.048	0.054	0.5
retrofit B	1.17160	+20.01	5.828	0.142	0.159	1.4
retrofit C	1.17166	+26.13	5.826	0.185	0.207	1.8

falls short of the estimate when COP rises with load. The plant's operating point is described by its COP (33) and its load as a fraction of rated capacity, read against the efficient band. The robustness of the PUE pick is measured by the margin g of (43), in PUE units, read against zero: its sign in each combination says whether the pick survives, and its size against the retrofits' own PUE changes says how close the ranking is to turning. The network's solve is checked against the closed form (14), whose reference is round-off.

4 Results

4.1 Baseline operating point

At baseline the campus draws 37.68 kW of computing power, 12.48 kW in building A, 12.58 kW in building B, 12.62 kW in building C. Per rack that is 416.1, 419.3, 420.7 W against a base power of 400 W: the rest is the temperature-driven part of (2), largest in the worst-contained building, and the values are those of the closed form (14). The plant carries 62.8 % of its rated capacity with two units running, each at the same fraction of its own capacity, well inside the efficient band that starts at 28.3 %. It draws 6.46 kW at a COP of 5.834, below the 6.251 it reaches at full load on the same water, and the campus PUE is 1.17140, which is $1 + 1/\mathrm{COP}$ as (35) requires.

4.2 Energy saved by each retrofit

Figure 3 shows the site power of the four scenarios. Panel (a) stacks computing power (dark) and chiller power (light) for each scenario on a common axis from zero, with the total above each bar; panel (b) shows each retrofit's saving against the baseline split into the part saved at the racks (dark) and the part saved at the chiller (light). Table 1 gives the same comparison as numbers, with the PUE and the chiller's COP of every scenario. Retrofit C saves 0.21 kW at the site, 1.8 MWh/yr, against 1.4 MWh/yr for retrofit B and 0.5 MWh/yr for retrofit A; all three save energy against the reference of no saving, and retrofit C is the recommendation. The order follows from (14): a retrofitted building's racks all fall to the same 414.5 W, so a retrofit saves the baseline excess of its building over that level, 1.6 W, 4.7 W, 6.2 W per rack in buildings A, B, C, which is largest where containment is worst. The computing power saved alone ranks the retrofits in the same order; the chiller adds to each saving.

4.3 Campus PUE under the retrofits

Every retrofit raises campus PUE (Table 1): from 1.17140 to 1.17146 for retrofit A, 1.17160 for retrofit B, 1.17166 for retrofit C. A lowest-PUE rule therefore picks retrofit A, which saves the least energy, 0.5 MWh/yr, and ranks retrofit C, which saves the most, last. By (35) PUE rises exactly

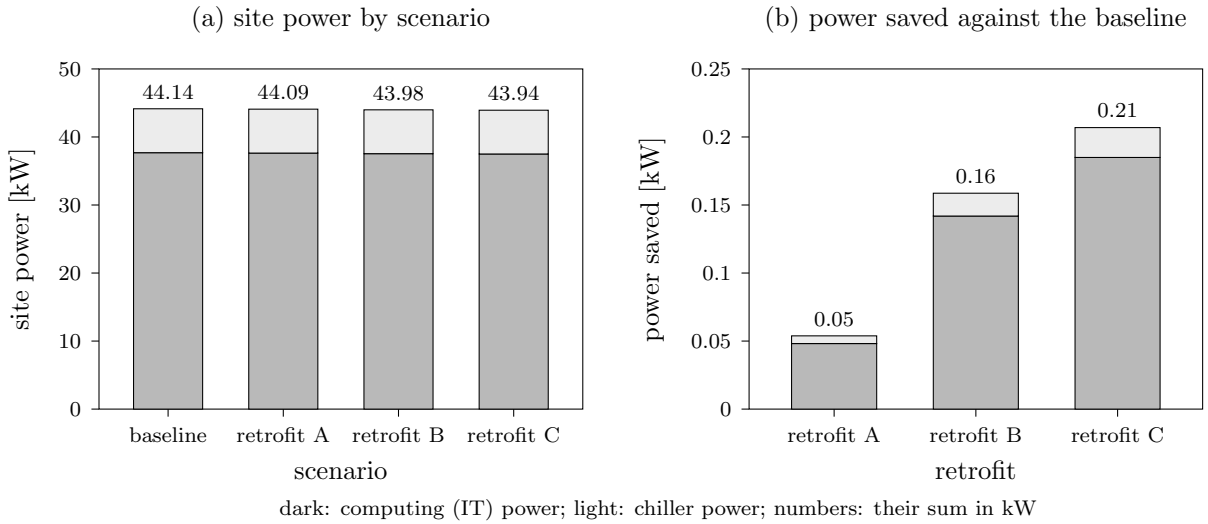


Figure 3: Site power of the four scenarios. (a) Computing power and chiller power stacked, for the baseline and for the retrofit of each building. (b) The part of each retrofit's saving that comes from the racks and the part that comes from the chiller. The retrofit of building C saves the most, 0.21 kW at the site, of which 0.02 kW at the chiller.

because the chiller's COP falls, from 5.834 to 5.832, 5.828, 5.826. Figure 4 shows why. Panel (a) plots the plant's COP against its load as a fraction of rated capacity over the whole range, with the lower edge of the efficient band dashed, the load at which the second unit is staged on dotted, and the four scenarios as dots; panel (b) enlarges the neighborhood of the four dots. Every scenario runs two units, each between 62.5 and 62.8 % of its capacity, on the part of the curve where COP rises with load toward its full-load value. A retrofit lowers the duty (32), the plant moves down that curve and loses efficiency: retrofit C cuts computing power by 0.18 kW (0.5 %) but chiller power by only 0.022 kW (0.3 %), so the chiller's share of the site's power, and with it PUE, grows although the site draws less. The retrofit that saves the most moves the plant furthest and raises PUE most.

4.4 Site power saved per watt saved at the racks

For retrofit C the site saves $\rho = 1.118$ W per W saved at the racks (39), and the other two retrofits give 1.119 and 1.118. The site saves 1.12 W per W saved at the racks, which the constant-COP estimate $\rho_{\text{const}} = 1.171$ overstates, because COP moves with load: the estimate charges the removed rack power at the baseline COP, while (41) subtracts the efficiency the plant loses as its load falls. The estimate equals the baseline PUE by (35); estimating a retrofit's site saving as its rack saving times the baseline PUE would overstate it by 4.7 %.

4.5 Rack temperatures

Figure 5 plots the temperature of each of the 90 racks, grouped by building, at baseline (open gray circles) and after the retrofit of building C (black dots). Building C's racks fall from 313.8 K to 309.7 K as its cold aisle falls from 301.2 K to 297.3 K, while the racks of the other two buildings keep their temperatures. The buildings do not interact because each one's heat balance closes on the chilled-water temperature (12); their only coupling is through the plant, which prices their

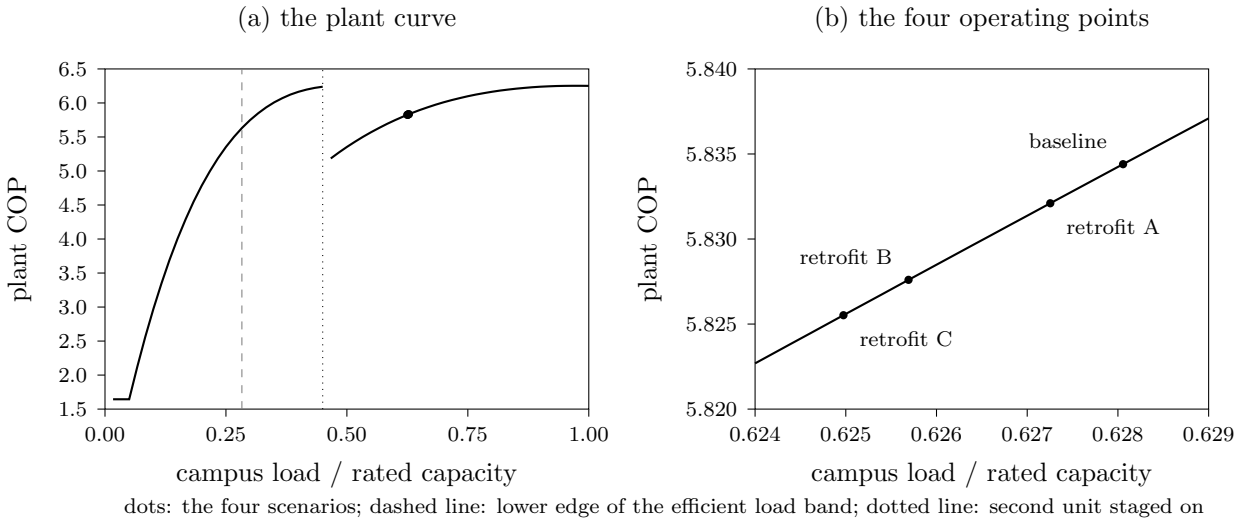


Figure 4: Plant COP against campus load at the 85°F condenser heat sink. (a) The whole range: one unit runs below 0.45 of rated capacity and two above, and COP rises with each unit's load up to 6.25 at full load on the operating chilled water. (b) The neighborhood of the four scenarios: every retrofit lowers the load and moves the plant down its curve, the recommended one furthest.

summed heat but does not change their temperatures.

Four checks test that the solved network is physically plausible, over all 360 rack values of the four scenarios. The racks run between 309.69 and 313.80 K, below a bound of 320 K; the coldest cold aisle is 297.26 K, above a bound of 291.15 K, so no rack takes in air colder than the room can supply; the temperature-driven part of rack power is at most 5.2 % of the base power, below 10.0 %; and the heat the buildings reject into the chilled water equals the computing power the plant is charged for to a relative difference below 10^{-9}. Every check passes. The plant is also checked against the range of the chiller law's calibration: in all 48 plants of the survival check and at every scenario's load, no operating point leaves it, with the chilled water entering between 287.18 and 289.26 K.

4.6 Survival of the lowest-PUE pick

Table 2 lists, for the registered plant, each single change of plant choice and the joint worst of the 48 combinations, the other retrofits' PUE minus that of retrofit A and the margin (43). The lowest-PUE pick holds under every one of the six single changes, with margins from +4.54 to $+18.42 \times 10^{-5}$ PUE, and holds even at the joint worst of 48 combinations, loss scale 0.8, four units of 15 kW, where the margin is $+3.81 \times 10^{-5}$. The margins of the single changes are of the order of the retrofits' own PUE changes, of the order of 10^{-4}, so the PUE ranking rests on a small slope of the plant curve, while the energy ranking is set by the racks. The recommendation rests on energy saved, not on PUE.

5 Discussion

The energy ranking and the PUE ranking disagree because PUE divides by the quantity the retrofit changes. When a decision lowers computing power, a campus PUE can rise while the bill falls, and on this plant it does for every retrofit; Brady et al. (2013) caution for this reason against reading

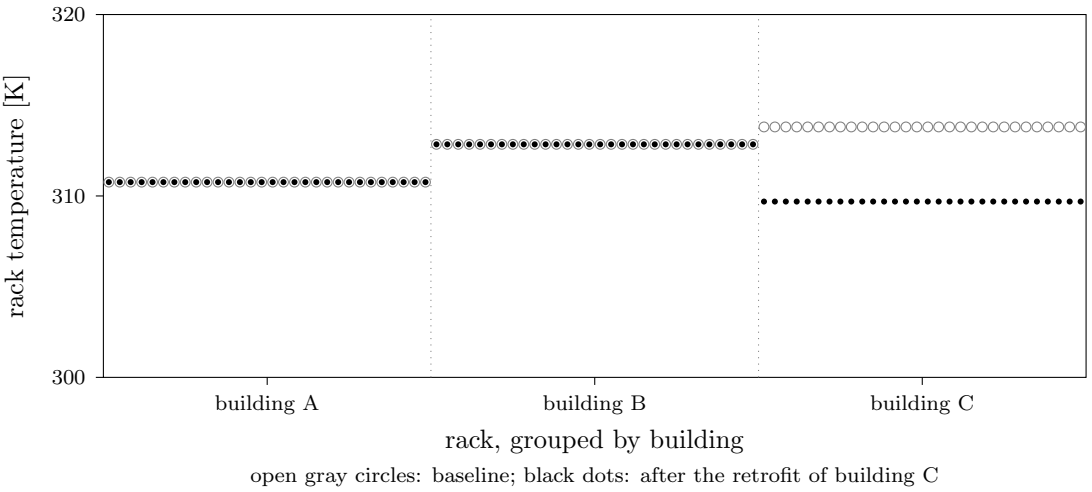


Figure 5: Temperature of each of the 90 racks before and after the retrofit of building C. Racks in one building share one temperature because their rows and rooms are identical. The retrofitted building's racks fall from 313.8 K to 309.7 K; the other buildings' racks do not move.

PUE as a measure of overall energy efficiency. PUE remains a fair comparison between plants at a fixed computing load, which is not the comparison a containment retrofit poses.

The results stop where the model stops. The model is steady and assumes the solved operating point for the whole year, with no daily or seasonal variation of workload. The plant sees a fixed 85°F condenser in every hour, while the IPLV credit its units are anchored to assumes 75/65°F condenser water at part load; a plant that sees cooler condenser water would draw less and could have a different slope of COP with load. The chiller law keeps the shape measured on one centrifugal chiller and is extrapolated by similarity to positive-displacement units of 8.5 tons that the measurement does not cover. Staged unloading of the compressors and the losses of short cycling are not represented; below its minimum stable load a unit's power is a linear time average. The plant is priced at the water the buildings reject their heat to, 61°F entering, which is 7°F warmer than the rating point at which the units are anchored to the code minimums; the law credits the warmer water with a higher efficiency, a full-load COP of 6.25 against 5.58 at the rating point, so a unit that just meets the code as rated runs better than the code's figure here. The survival check shows that the PUE ranking holds under every combination of these choices, but by margins no larger than the retrofits' own PUE changes; it does not show that the energy ranking survives every plant, but the computing power saved alone already ranks the retrofits in the recommended order.

The thermal network's conductances follow a design rule, not measured airflow: a 12 K rise across the racks and 3 K coil approaches at the base rack power. They put the racks between 309.7 and 313.8 K and the cold aisles of the two worse-contained buildings at 300.3 K and above, toward the warm end of the inlet temperatures data halls are run at, and the temperature dependence of rack power is linear with one coefficient for every rack. The absolute savings therefore follow the sizing rule and the leakage coefficient; what carries over is the mechanism, that recirculation raises rack power in proportion to the mixing term of (13) and that a plant whose COP rises with load returns less than 1/COP of each rack watt saved. The retrofit's cost is not modeled, so the report ranks energy, not return on investment.

An earlier version of this study used hand-set conductances ($G_{\mathrm{rk}} = 15$, $G_{\mathrm{s}} = G_{\mathrm{ret}} = 50$,

Table 2: Survival of the lowest-PUE pick, retrofit A, across the registered chiller plant choices. Each row re-prices the four scenarios' solved computing loads with a plant differing from the registered one in the listed choices: the registered plant, each single change, and the joint worst of all 48 combinations. The middle columns are the other retrofits' PUE minus the pick's; the margin is the smaller of the two; a negative margin means another retrofit has the lowest PUE.

plant choice changed	$\mathrm{PUE_B} - \mathrm{PUE_A}$ $[10^{-5}]$	$\mathrm{PUE_C} - \mathrm{PUE_A}$ $[10^{-5}]$	margin g $[10^{-5}]$	pick
none (the registered plant)	+13.25	+19.37	+13.25	holds
loss scale 0.8	+10.70	+15.64	+10.70	holds
loss scale 1.2	+15.75	+23.03	+15.75	holds
least-power staging	+13.25	+19.37	+13.25	holds
four units of 15 kW	+4.54	+6.65	+4.54	holds
condenser floor 65°F	+13.25	+19.37	+13.25	holds
constant-flow chilled water	+18.42	+26.93	+18.42	holds
joint worst of 48: loss scale 0.8, four units of 15 kW	+3.81	+5.58	+3.81	holds

$G_{\mathrm{room}} = 500$ and $G_{\mathrm{bldg}} = 3000$ W/K), a 290 K boundary uncoupled from a plant priced at 54°F entering water, and rack nodes up to 389 K. It reported savings of 17.97, 13.45, 4.33 MWh/yr for retrofits C, B, A, a baseline of 46.21 kW of computing power at a PUE of 1.1831, and a lowest-PUE pick that flipped at the joint worst plant (margin -0.0118). The overheated racks inflated the temperature-driven part of rack power, and with it the savings; with conductances sized to the racks' heat the savings fall by a factor of about 10, the order of the retrofits and the disagreement between the energy and PUE rankings are unchanged, and the pick holds under every plant.

6 Conclusion

This report asked which of three buildings' containment retrofits saves the most electricity on a campus whose buildings share one chilled-water plant of two code-minimum chiller units, and whether choosing the retrofit with the lowest campus PUE gives the same answer. Retrofitting the worst-contained building, C, saves the most, 1.8 MWh/yr at the campus meter, against 1.4 MWh/yr for B and 0.5 MWh/yr for A, starting from a baseline of 37.68 kW of computing power, 6.46 kW of chiller power and a PUE of 1.17140. The lowest-PUE rule does not agree: every retrofit raises PUE, to 1.17146, 1.17160, 1.17166 for retrofits A, B, C, so it picks A, the smallest saver. The reason is the plant: its COP falls from 5.834 to 5.826 as the retrofit lowers its load, so chiller power falls proportionally less than computing power. For the same reason the site saves 1.12 W per W saved at the racks, which the constant-COP estimate of 1.17 overstates. The lowest-PUE pick holds under each single change to the plant model and in all 48 combinations, by margins of the order of 10^{-4} PUE; either way the recommendation rests on energy saved, not on PUE. The next step is to price the same retrofits with a plant that sees hourly weather and with conductances derived from measured airflow.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level0_site/campus_pue_retrofit_ranking
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
cd ../../../../..
```

```
python -m pytest -q tests/plugins/datacenter/test_campus_pue_retrofit_ranking_case_study.py
```

`run.py` solves the four scenarios, prices them, runs the survival check over the 48 plant combinations and writes `results.json`; `make_report.py` reads only `results.json` and writes `report.tex`. No random numbers are drawn. The test in the last line regenerates the results, compares them with the committed file and rebuilds the tex.

B Data and problems

No external data are used; the campus is the study's own construction. The chiller law's coefficients are the RP-1043 fit carried in the repository's chiller-plant model, and the efficiency minimums are those of ASHRAE 90.1-2019, Table 6.8.1-3, Path A, for water-cooled positive-displacement chillers. The study leaves no corpus problems.

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