Evening peak reduction on a two-building data-center campus with a scheduled battery

Ricardo Marquez, Ph.D.
rmarquez123@gmail.com

September 2026

Abstract

A data center draws a large electrical load around the clock, and a tariff that bills the highest hour of the month makes a few hours of evening peak expensive. A battery that charges at night and discharges through the peak can lower the billed maximum, provided it holds enough energy for the whole peak after its conversion losses and stays inside its state-of-charge and temperature limits. This report evaluates one such schedule on a two-building campus whose 48 kW computing load rises by 50 % between 16:00 and 21:00. The campus is written as a steady thermal network of racks, aisles, rooms and buildings on a shared chilled-water plant priced by the Gordon–Ng chiller law anchored to the ASHRAE 90.1 minimum efficiencies, and a 220 kWh battery with charge and discharge efficiencies of 0.95 and 0.90 and a lumped heat capacity is integrated through the day, charging at 5 kW for 6 hours and discharging at 25 kW for 5. Solved hour by hour, the campus peaks at 83.81 kW in the evening with a power usage effectiveness of 1.162. With the battery the evening import falls to 58.81 kW, below the daytime import of 59.43 kW, so the daytime plateau becomes the billed peak and the grid peak falls by 24.38 kW, less than the 25 kW rating: the last 0.62 kW of the evening discharge buys nothing. The battery ends the day at 16 % state of charge, 1.8 kWh above its 15 % floor, peaks at 310.3 K (37.2 °C) against a 318 K limit, and converts 15.4 kWh into heat; the state-of-charge floor, not the temperature limit, bounds the schedule. Across 48 combinations of chiller-plant modeling choices the evening stays fully absorbed in all of them, and at least 24 kW of the schedule lands on the peak in 40; it falls short, by 0.23 kW, in the combinations that pair a 20 % more efficient plant with constant-flow chilled water. The study is a forward one: every input is stated and nothing is estimated from data. The computation takes 24 steady network solves and 24 one-hour battery integrations.

1 Introduction

Data centers convert nearly all the electricity they draw into heat, and removing that heat adds a cooling load on top of the computing load; the ratio of the two, the power usage effectiveness, is the standard measure of how much a facility spends on everything but computing (Barroso et al., 2019). Commercial tariffs bill both the energy a site uses and a demand charge on its highest metered interval, so a load that peaks for a few hours sets a large part of the bill and also the capacity the utility must hold for the site. Where the computing load follows its users into the evening, a battery on the customer side of the meter can shave that peak: it charges while the load is low and discharges while it is high, and the grid sees the difference. Whether a given battery and schedule do so is not a matter of subtraction alone. The battery loses part of every kilowatt-hour it stores, it must last the whole peak without falling below its minimum state of charge, its losses

heat it toward its temperature limit, and the site load it shaves includes a chiller plant whose power depends on the load it carries through its part-load efficiency.

Sizing a battery and choosing its operation for peak shaving is an established problem; Oudalov, Cherkaoui and Beguin (2007) size a battery energy storage system and optimize its operation for the peak-shaving application. The cooling side rests on thermodynamic chiller models: Chua, Ng and Gordon (1996) relate measured chiller behavior to the thermodynamic model of Gordon and Ng, whose universal form this report uses for the plant's compressor power, and the minimum efficiencies a chiller must meet at full and part load are fixed by ASHRAE Standard 90.1 (ASHRAE, 2019). The power and cooling structure of a data center and the role of its power usage effectiveness are described by Barroso, Hölzle and Ranganathan (2019). This report adds nothing to those methods; it applies them together to one campus and one schedule.

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses its data-center plugin to write the campus's rack, aisle, room and building heat balances, its battery-storage plugin to integrate the battery's stored energy and temperature through the day, and its chiller-plant model to price the cooling; the two plugins share nothing but the site's power balance. Section 3 states the framework and each of these constructs precisely.

This report checks one fixed schedule: whether a 220 kWh battery that charges at 5 kW from 00:00 to 06:00 and discharges at 25 kW from 16:00 to 21:00 cuts the grid peak of a two-building data-center campus, and whether it stays inside its charge and temperature limits while doing so. It solves the campus's thermal network and chiller plant for every hour of one synthetic day, integrates the battery through the same day, and combines them at the site meter; it then repeats the plant pricing across 48 combinations of the plant's modeling choices to see whether the reduction depends on them. It is a forward study: every input is stated and nothing is estimated from data. Section 2 describes the campus, the battery and the model, Section 3 the equations and the protocol, Section 4 the results, Section 5 their limits, and Section 6 the answers.

2 System and data

The campus. Figure 1 draws the site. Two buildings share one chilled-water plant and one connection to the utility grid. The campus computing load is 48 kW scaled by an hourly multiplier; Building A carries 56 % of it and Building B 44 %. Each building has 2 rooms, each room one row of 4 racks with a cold aisle the racks draw air from and a hot aisle they exhaust into, so the campus has 16 racks and each building's share is split equally among its 8. The multiplier is 0.88 from 00:00 to 06:00, 1.00 from 06:00 to 08:00 and from 21:00 to 24:00, 1.05 from 08:00 to 16:00, and 1.50 from 16:00 to 21:00, the evening peak. The power usage effectiveness (PUE) is the site's electrical power divided by the computing power; here the site's power is the computing load plus the chiller compressors.

The chiller plant. The plant has 2 identical water-cooled positive-displacement units of 90 kW (25.6 tons of refrigeration) each, 180 kW in all. Each unit is the least efficient machine that meets both ASHRAE 90.1 minimum efficiencies for its size: 0.750 kW of compressor power per ton of cooling at full load and 0.600 kW per ton on the integrated part-load value (IPLV), a

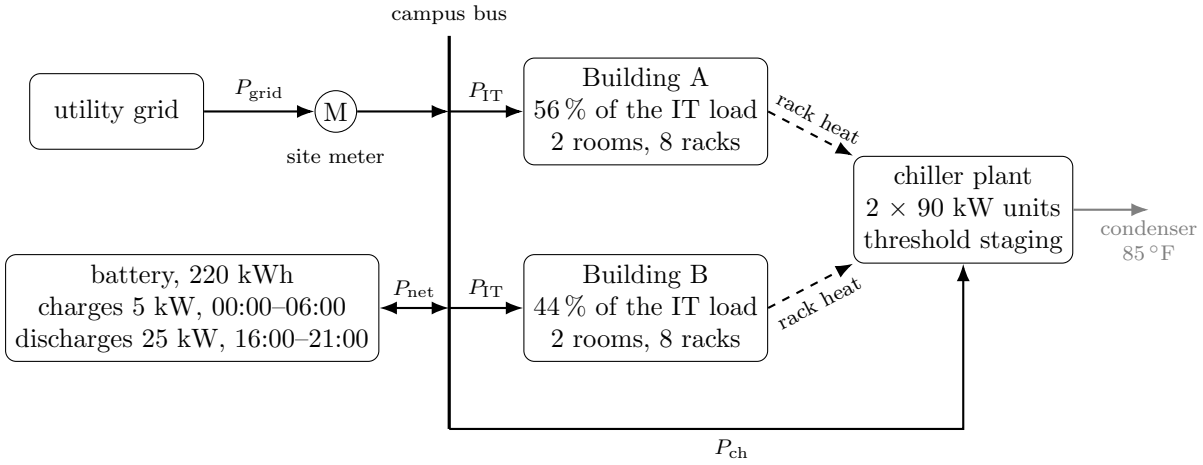


Figure 1: The site of the study. The utility grid supplies the campus through the site meter, which reads the grid import P_{grid}. On the campus bus two buildings of racks draw the IT load P_{IT}; the chiller plant draws the compressor power P_{ch} that removes the racks' heat, carried to it in chilled water (dashed) and rejected to condenser water; and a 220 kWh battery charges from the bus overnight and discharges into it through the evening peak, its net output P_{net} positive when discharging. The quantity of interest is the day's highest hourly P_{grid}.

weighted average over four part loads. The condenser sees water at 85 °F, the AHRI 550/590 rating temperature, all day; chilled water enters the units at 61 °F and leaves at 51 °F, the same water the campus rejects its heat into, since the network's chilled-water boundary is held at the units' entering temperature; and units are brought on when the running ones would exceed 90 % of their capacity. Each unit's efficiency level is set against the ASHRAE minimums at the AHRI rating point, 54 °F entering, so the warmer operating water earns the efficiency it buys. The entering temperature, 61 °F, is the warmest whole degree inside the range the chiller law is calibrated on.

The battery and its schedule. The battery stores 220 kWh at full charge and starts the day at a state of charge of 0.66, the fraction of that capacity it holds; it may operate between 0.15 and 0.95. Its rated power limits are 40 kW charging and 60 kW discharging. Of the power sent to charge it, a fraction 0.95 is stored; of the stored energy drawn to discharge it, a fraction 0.90 is delivered; the rest becomes heat in the battery. The battery has a heat capacity of 4.0 MJ/K, loses heat to the ambient air at 90 W/K, starts at 299 K and may not exceed 318 K. The ambient temperature follows a daily sine of 3 K amplitude about 299 K, highest at 14:00. The schedule charges at 5 kW from 00:00 to 06:00, discharges at 25 kW from 16:00 to 21:00, and rests otherwise; both powers are well inside the rated limits.

The data. No measured data are used. The load profile, the campus geometry and conductances, the battery and its schedule, and the ambient temperature are the study's own construction and are stated in full here and in Section 3; the chiller law's shape comes from coefficients fitted to one laboratory chiller, stated with the law. The computation is deterministic: there is no noise and no random draw.

Assumptions.

- Units are SI internally; powers are reported in kW, energies in kWh, temperatures in K with Fahrenheit for the chiller rating conditions.
- Every hour of the campus is a settled steady state at that hour's load; the battery is integrated in time, its state carried from hour to hour.
- The thermal network and the chiller plant share one chilled water: the network's boundary is held at 289.26 K, the 61 °F water entering the units; the condenser water is at 85 °F, with no weather.
- The chiller duty equals the campus IT power: in steady state every watt the racks draw leaves through the chilled water. Fans, pumps, cooling towers and power conversion are not counted in the site power.
- The battery is connected on the campus side of the meter without electrical losses outside its own efficiencies; its heat goes to the ambient air, not to the campus's cooling.
- The schedule is fixed in advance; the battery's rated power limits are not enforced by the model and are not approached.

The model. Figure 2 shows the coupled model. The campus unknowns are the temperatures of every rack, aisle, room and building node, the heat flows between them and every rack's electrical power; conservation at each node and a conductance law on each connection tie them, and the solved rack powers set the plant's duty. The battery's unknowns are its stored energy and temperature, tied by energy and heat balances to the scheduled powers and the ambient air. The two meet only in the site balance.

The questions. (1) Does the scheduled battery reduce the campus's highest hourly grid import, and by how much? (2) Does the battery stay inside its state-of-charge and temperature limits through the day? (3) Does the reduction depend on how the chiller plant is modeled?

3 Methods

3.1 The campus thermal network

Each rack i dissipates its electrical power as heat, and its power grows linearly with its temperature, the leakage of its electronics. Its base power P_0 is its building's share s_B of the campus load $\bar{P} = 48$ kW times the hour's multiplier $m(h)$, divided among the building's 8 racks. The rack rejects its heat into the hot aisle in proportion to its temperature above the cold aisle; the room's air supply, the recirculation of hot air into the cold aisle and the return air to the room are conductances between the aisle and room nodes; the room rejects its heat to the building node and the building to the chilled water. With temperatures T in kelvin and heat flows Q in watts, the constitutive

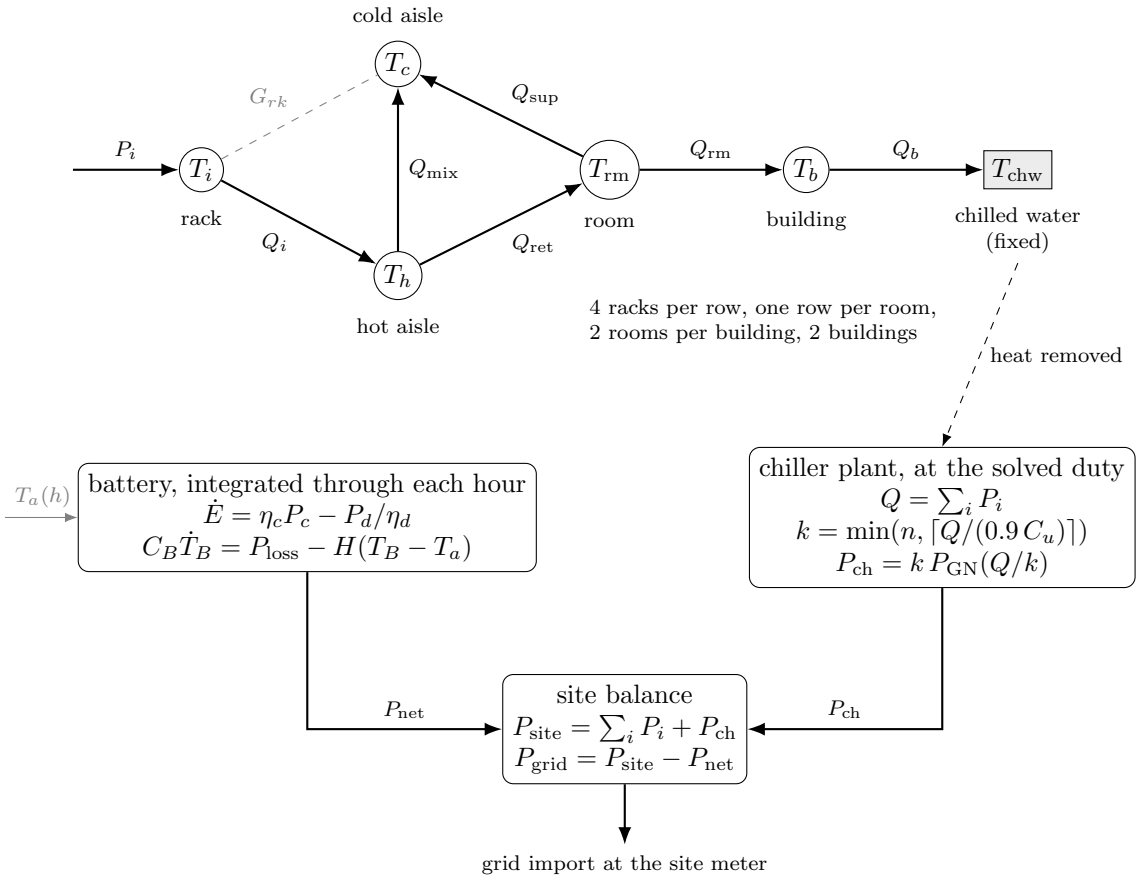


Figure 2: The coupled model. Top: the steady thermal network of one rack i, its row's cold and hot aisles, its room and its building, with every heat flow an arrow and the chilled-water temperature a fixed boundary. The dashed line marks the rack-over-cold-aisle temperature difference that drives Q_i; every node balances the flows that meet at it, and the rack's electrical power P_i grows with its temperature. Bottom: the chiller plant is evaluated after the network is solved, at the duty Q equal to the campus IT power; the battery's stored energy E and temperature T_B are integrated through each hour under its schedule; the site balance combines both into the grid import. The battery does not feed back into the campus.

relations are

$$P_0 = s_B\, \bar{P}\, m(h)/8, \tag{1}$$
$$P_i = P_0 + \alpha\, (T_i - T_{\text{ref}}), \tag{2}$$
$$Q_i = G_{rk}\, (T_i - T_c), \tag{3}$$
$$Q_{\text{sup}} = G_s\, (T_{\text{rm}} - T_c), \tag{4}$$
$$Q_{\text{mix}} = \lambda\, G_s\, (T_h - T_c), \tag{5}$$
$$Q_{\text{ret}} = G_r\, (T_h - T_{\text{rm}}), \tag{6}$$
$$Q_{\text{rm}} = G_{\text{rm}}\, (T_{\text{rm}} - T_b), \tag{7}$$
$$Q_b = G_b\, (T_b - T_{\text{chw}}), \tag{8}$$

and the conservation residuals, one per node, are

$$r_i = P_i - Q_i, \tag{9}$$
$$r_c = Q_{\text{sup}} + Q_{\text{mix}}, \tag{10}$$
$$r_h = \sum_{i \in \text{row}} Q_i - Q_{\text{mix}} - Q_{\text{ret}}, \tag{11}$$
$$r_{\text{rm}} = Q_{\text{ret}} - Q_{\text{sup}} - Q_{\text{rm}}, \tag{12}$$
$$r_b = \sum_{\text{rooms}} Q_{\text{rm}} - Q_b. \tag{13}$$

The constants are $\alpha = 0.8$ W/K and $T_{\text{ref}} = 300$ K in (2); $G_{rk} = 420$ W/K, $G_s = 1,680$ W/K, $G_r = 1,680$ W/K and the mixing factor $\lambda = 1.2$ in the row; $G_{\text{rm}} = 6,720$ W/K; $G_b = 13,440$ W/K; and $T_{\text{chw}} = 289.26$ K, a fixed boundary equal to the chiller units' entering water. The conductances follow one sizing rule: each link carries its design heat, that of the heaviest rack the study evaluates (Building A at the evening peak, 5.04 kW), at a stated temperature difference, a 12 K rise across the rack's air stream (G_{rk} and, for the row's 4 racks, $G_s = G_r$) and 3 K approaches at the room and building coils (G_{rm}, G_b). The cold-aisle balance (10) has no rack term because the racks read the cold-aisle temperature in (3) but deliver their heat to the hot aisle in (11). Summed over a building, the balances make Q_b equal to the building's rack power, so in steady state the chilled water removes exactly the IT power. The residuals have as many equations as unknowns and are solved for every hour by Newton's method.

3.2 The battery

The battery has two storage states, its stored energy E and its temperature T_B. Its charge power P_c and discharge power P_d are fixed by the schedule within each hour and the ambient temperature T_a is fixed per hour. The charge edge adds $\eta_c P_c$ to the stored energy and the discharge edge removes P_d/η_d; the shortfall of both becomes loss heat, which warms the battery's heat capacity C_B against a loss to the ambient air through the conductance H:

$$\frac{dE}{dt} = \eta_c P_c - P_d/\eta_d, \tag{14}$$
$$\text{SOC} = E/C, \tag{15}$$
$$P_{\text{net}} = P_d - P_c, \tag{16}$$
$$P_{\text{loss}} = (1 - \eta_c)\, P_c + (1/\eta_d - 1)\, P_d, \tag{17}$$
$$C_B \frac{dT_B}{dt} = P_{\text{loss}} - H\,(T_B - T_a), \tag{18}$$
$$T_a(h) = \bar{T}_a + A \sin\big(2\pi(h - 8)/24\big). \tag{19}$$

The constants are $C = 220$ kWh, $\eta_c = 0.95$, $\eta_d = 0.90$, $C_B = 4.0$ MJ/K, $H = 90$ W/K, $\bar{T}_a = 299$ K and $A = 3$ K, with h the hour of the day starting at zero. The day starts from SOC = 0.66 and $T_B = 299$ K. Each hour is integrated over 3,600 s by the second-order backward differentiation formula starting from a 300 s step, from the state the previous hour ended in. Within an hour (14) has a constant right-hand side, so the stored energy changes linearly and exactly, and (18) relaxes toward $T_a + P_{\text{loss}}/H$ with the time constant $C_B/H = 12.3$ h.

3.3 The network model

The residuals of the campus network and the storage relations of the battery are two instances of one general construction, in which the unknowns of a network and the relations they must satisfy are written on its graph and solved together; this subsection states it. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{20}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{21}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in this report is estimated from data: every input is stated, and the results are the solutions of these relations and quantities computed from them. Everything that follows is a choice of unknowns and relations on this graph.

3.4 The chiller plant

The plant is not a factor of the network: with the chilled-water temperature fixed, its compressor power is a function of the solved duty, evaluated after the solve. The duty is the campus IT power, the running units k follow threshold staging with unit capacity $C_u = 90$ kW and $n = 2$ installed units, and the units share the duty equally:

$$Q = \sum_i P_i, \tag{22}$$

$$k = \min\left(n, \lceil Q/(0.9\,C_u) \rceil\right), \tag{23}$$

$$P_{\text{ch}} = k\, P_{\text{GN}}(Q/k). \tag{24}$$

One unit's compressor power at load q follows the universal form of the Gordon–Ng law, the thermodynamic chiller model whose experimental basis Chua et al. (1996) describe. With T_{chi} the chilled water and T_{ci} the condenser water entering the unit, and three loss coefficients — b_1 the internal entropy generation, b_2 an equivalent heat leak, b_3 the heat exchangers' total thermal resistance —

$$P_{\text{GN}}(q) = \frac{T_{\text{ci}}\, q + b_1 T_{\text{chi}} T_{\text{ci}} + b_2 (T_{\text{ci}} - T_{\text{chi}})}{T_{\text{chi}} - b_3\, q} - q, \tag{25}$$

$$P_{\text{GN}}^{\text{cyc}}(q) = \frac{q}{q_{\min}}\, P_{\text{GN}}(q_{\min}), \tag{26}$$

$$T_{\text{ci}} = \max(T_{\text{sink}}, T_{\text{floor}}), \tag{27}$$

$$T_{\text{chi}} = 61\,^{\circ}\text{F}. \tag{28}$$

Equation (25) holds at or above the minimum stable load $q_{\min} = 9$ kW; below it the unit cycles and its time-averaged power is (26). The condenser water T_{sink} is 85 °F (302.59 K) and the head-pressure floor T_{floor} is 55 °F. With $b_1 = b_2 = b_3 = 0$ the law is the Carnot limit; the b_3 term makes efficiency peak at part load.

The coefficients carry the shape measured on one 316.5 kW (90-ton) centrifugal laboratory chiller, $b_1^{\text{ref}} = 78.4267$ W/K, $b_2^{\text{ref}} = 121,971$ W and $b_3^{\text{ref}} = 7.71109 \times 10^{-5}$ K/W at capacity C_{ref}, scaled to the unit's capacity by similarity (entropy generation and heat leak grow with capacity, exchanger resistance shrinks with it) and multiplied by one loss factor s that sets the efficiency level:

$$b_1 = s\, b_1^{\text{ref}}\, C_u / C_{\text{ref}}, \tag{29}$$

$$b_2 = s\, b_2^{\text{ref}}\, C_u / C_{\text{ref}}, \tag{30}$$

$$b_3 = s\, b_3^{\text{ref}}\, C_{\text{ref}} / C_u, \tag{31}$$

$$\text{IPLV} = 0.01\, A + 0.42\, B + 0.45\, C + 0.12\, D, \tag{32}$$

$$s = \min(s_{\text{FL}}, s_{\text{IPLV}}). \tag{33}$$

Here s_{FL} is the factor at which the unit's full-load efficiency at the AHRI 550/590 rating point (54 °F chilled water entering, 85 °F condenser water) equals the ASHRAE 90.1 full-load minimum, a coefficient of performance (COP, heat removed per unit of compressor power) of 4.69, and s_{IPLV} the factor at which (32) equals the IPLV minimum, 5.86. In (32) A, B, C and D are the COP at 100, 75, 50 and 25 % load with condenser water at 85, 75, 65, 65 °F and chilled water leaving at 44 °F under constant flow, the floor applied at each point. COP falls as s grows, so each factor is found by bisection, and the smaller one (33) gives the least efficient unit meeting both minimums. For this unit the IPLV minimum binds: $s = 0.597$, giving $b_1 = 13.32$ W/K, $b_2 = 20,720$ W and $b_3 = 1.620 \times 10^{-4}$ K/W, an IPLV of 5.86 and a full-load COP of 5.58 at the rating point, above the full-load minimum of 4.69.

3.5 The site balance and the dispatch schedule

For every hour h the site power is the IT power plus the chiller power, the grid supplies the site power less the battery's net output (16), and the peak reduction compares the day's highest hour with and without the battery:

$$P_{\text{site}}(h) = \sum_i P_i(h) + P_{\text{ch}}(h), \tag{34}$$

$$\text{PUE}(h) = P_{\text{site}}(h) \Big/ \sum_i P_i(h), \tag{35}$$

$$P_{\text{grid}}(h) = P_{\text{site}}(h) - P_{\text{net}}(h), \tag{36}$$

$$\Delta P = \max_h P_{\text{site}}(h) - \max_h P_{\text{grid}}(h). \tag{37}$$

The schedule sets $P_c = 5$ kW for $h = 0, \ldots, 5$ and $P_d = 25$ kW for $h = 16, \ldots, 20$, and both to zero otherwise; hour h covers $[h, h+1)$ and is reported as hour ending $h+1$, so the discharge window W is the hours ending 17–21. The campus is solved and priced once per hour, the battery integrated once per hour, and (36) combines them; the battery does not change the campus. When several hours share the highest grid import to within 1×10^{-6} kW they form a plateau, reported at its first hour.

3.6 Sensitivity to the plant modeling choices

The plant rests on choices a reader could make differently, so the verdict is put through a sensitivity analysis over those choices: the hourly chiller power is re-priced under every combination of five factors, with the campus IT power and the battery trajectory held fixed: the loss factor s multiplied by 0.8, 1, 1.2; threshold staging (23) or least-power staging, which runs the number of units that draws least among those that can carry the duty; the registered unit count or twice the units at half the size, each unit anchored again at its own size; a condenser-water floor of 55 or 65 °F, the unit anchored again under it; and chilled water entering at the fixed 61 °F (28) or under constant flow, entering at 51 °F plus 10 °F times the unit's load fraction. That makes 48 combinations. In each, the verdict has two clauses, both positive when it holds, and the margin is the smaller:

$$c_1 = \Delta P - 24.00 \text{ kW}, \tag{38}$$

$$c_2 = \max_{h \notin W} P_{\text{grid}}(h) - \max_{h \in W} P_{\text{grid}}(h), \tag{39}$$

$$g = \min(c_1, c_2). \tag{40}$$

The first clause asks whether most of the 25 kW schedule still lands on the peak; its threshold marks that decision rather than the rating. The second asks whether the evening is fully absorbed, so that the daytime import, which the battery does not serve, sets the bill. Because the campus peak lies in the window and the discharge is a flat 25 kW, $c_1 + c_2 = 1$ kW in every combination, so the second clause is not independent evidence. The verdict holds in a combination when $g \geq 0$; the joint worst combination is the one with the smallest g, ties ordered by the factors in the order listed and each factor's levels in the order given.

3.7 Metrics

The first question is answered by the peak reduction ΔP (37) in kW, the quantity a demand charge bills. Its reference is the discharge rating of 25 kW: a schedule whose discharge covers every peak hour and leaves the new peak inside the window reduces the peak by exactly its rating, a smaller value means the peak escaped the window or the battery ran out, and the gap between the campus peak and the highest import outside the window bounds how much any evening discharge could achieve. The second question is answered by the state of charge (15) at the end of every hour, read against its limits 0.15 and 0.95, with the stored energy above the floor in kWh as the margin; by the battery temperature against its 318 K limit, with the difference in K as the margin; and by the day's loss energy, the sum of P_{loss} (17) over the hours in kWh, read against the energy the schedule moves. The third question is answered by the margins c_1, c_2 and g (40) in kW, read against zero, and by the count of combinations in which the verdict holds, read against all 48. The PUE (35) at the peak hour describes the campus the battery serves. Cost is counted in solves and integrations rather than timed.

4 Results

Grid peak. Figure 3 plots the site power P_{site} (34) and the grid import P_{grid} (36) for each hour of the day as steps, with the battery's net output as bars below and the discharge window shaded; the two peak values are labeled at the right. Without the battery the campus peaks at 83.81 kW in each of the hours ending 17–21, with a PUE of 1.162. With it, the grid import in those hours falls to 58.81 kW, which is 0.62 kW below the daytime import of 59.43 kW in the hours ending 9–16. That daytime plateau becomes the highest hour of the day, reported at hour ending 9 (08:00–09:00). The

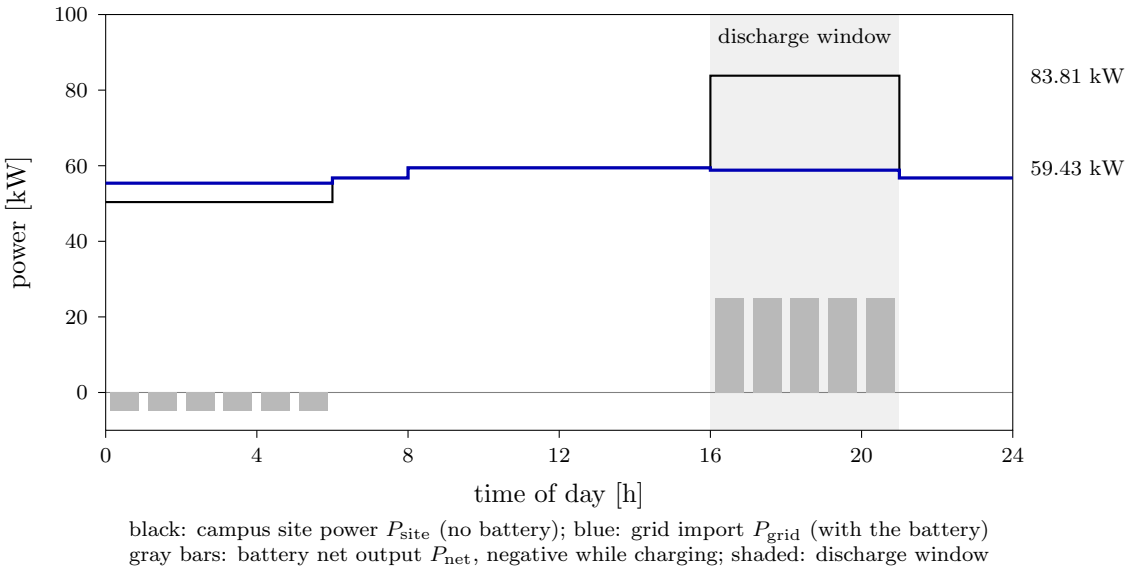
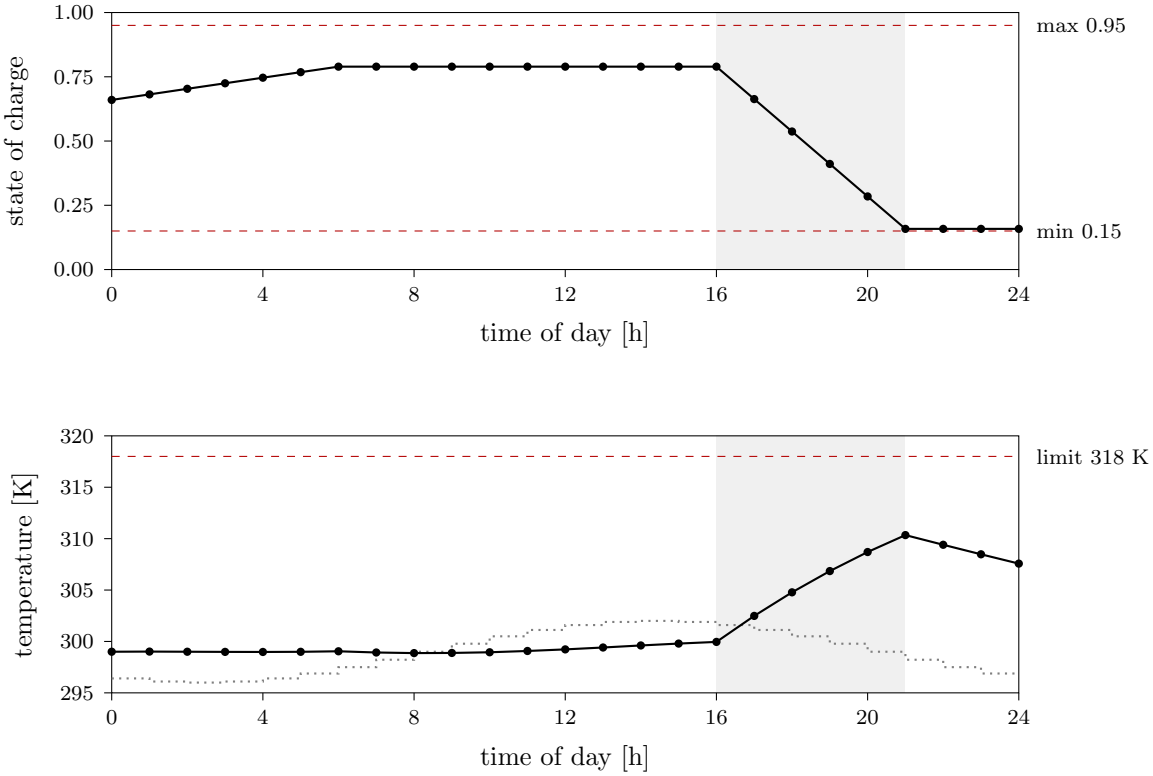


Figure 3: Campus site power and grid import over the day, one value per hour. The battery's 25 kW discharge between 16:00 and 21:00 lowers the evening import from 83.81 kW to 58.81 kW, below the daytime import of 59.43 kW, so the day's maximum falls to 59.43 kW over the 8 daytime hours ending 9–16; overnight the grid import runs 5 kW above the site power while the battery charges.

reduction (37) is $\Delta P = 24.38$ kW, less than the 25 kW discharge rating: the campus peak exceeds the highest hour outside the window by exactly 24.38 kW, no evening discharge can cut the billed peak by more, and the discharge overshoots it by 0.62 kW, a surplus that lowers no billed hour. Overnight the import is 55.38 kW, the site's 50.38 kW plus 5 kW of charging, below both.

Battery limits. Figure 4 plots the state of charge (15) and the battery temperature at the end of each hour, with their limits dashed, the ambient temperature (19) as dotted steps and the discharge window shaded. The state of charge rises by 0.0216 per charging hour to 0.790 at 06:00, well below the 0.95 ceiling, holds through the day, and falls by 0.126 per discharging hour, as (14) gives, to 0.158 at 21:00, where it stays. That is above the 0.15 floor by 1.81 kWh of the 34.81 kWh left, enough for about 4 more minutes of discharge at 25 kW: the floor is nearly reached, and it is the limit that bounds the schedule. The battery temperature stays within a kelvin of 299 K through the charging and rest hours, rises during the discharge, and peaks at 310.3 K (37.2 °C) at 21:00, 7.7 K below the 318 K limit, before relaxing to 307.6 K at midnight. Equation (18) explains the margin: a sustained 25 kW discharge loses 2.78 kW and would settle 30.9 K above ambient, beyond the limit, but with a time constant of 12.3 h the 5-hour discharge covers only 33 % of that rise, and the stored energy runs out long before the temperature could.

Energy and losses. The schedule draws 30 kWh from the grid to charge and delivers 125 kWh into the evening, the difference coming from the 173.7 kWh stored when the discharge begins. The loss (17) over the day is 15.39 kWh, 1.50 kWh while charging and 13.89 kWh while discharging, 12.3 % of the energy delivered: at $\eta_d = 0.90$ every delivered kilowatt-hour costs 1.111 kWh of stored energy.



dots: state of charge (top) and battery temperature T_B (bottom) at the end of each hour; dashed: limits
dotted steps: ambient temperature T_a; shaded: discharge window

Figure 4: Battery state through the day. Top: state of charge, starting from 0.66 at midnight, against its limits of 0.15 and 0.95. Bottom: battery temperature against its 318 K limit and the ambient temperature it rejects heat to. The state of charge ends at 0.158, 1.8 kWh of stored energy above the floor; the temperature peaks at 310.3 K at 21:00, 7.7 K below the limit.

The campus and its plant. Table 2 in Appendix C lists every hour. The chiller plant runs one unit all day: the largest duty, 72.12 kW, stays below the 81 kW at which (23) brings on a second unit. Its COP rises with load from 5.20 overnight to 6.17 at the peak, as the part-load behavior of (25) gives, so the chiller draws 8.13 to 11.70 kW and the PUE falls from 1.192 at night to 1.162 at the peak. Rack leakage (2) adds 0.01 kW to the nominal 42.24 kW overnight and 0.12 kW to the nominal 72.00 kW at the peak. The rack node temperatures run from 299.5 to 311.5 K and the coldest cold aisle is 294.0 K; over all 384 rack values of the day every rack stays at or below 320 K, every cold aisle at or above 291.15 K, the leakage within 10 % of each rack's base power, and the heat the buildings reject into the chilled water equals the rack load the plant is priced at to within 1×10^{-12} relative. Across every plant combination of the sensitivity analysis the chiller operates inside the range its law is calibrated on.

Sensitivity to plant choices. Table 1 lists the two clauses (38) and (39) and the margin (40) for the registered plant, for each plant factor changed alone, and for the joint worst of the 48 combinations. The verdict holds in 40 of 48. The second clause is at least 0.24 kW in every combination: the evening is absorbed whatever the plant. The first clause carries the information. It is 0.38 kW for the registered plant, 0.02 kW with the loss factor at 0.8, 0.12 kW with constant-flow chilled water, and -0.23 kW with both, the joint worst, where it fails in all 8 such combinations;

Table 1: Sensitivity of the peak reduction to the chiller plant's modeling choices. Each row reprices the chiller power of every hour with the campus IT load and the battery trajectory fixed: the registered plant, each factor changed alone, and the combination of all 48 with the smallest margin. Columns: c_1, the reduction above 24.00 kW; c_2, the highest grid import outside the discharge window above the highest inside it; the margin g, the smaller of the two; and whether the verdict holds ($g \geq 0$).

plant choice changed	c_1 [kW]	c_2 [kW]	g [kW]	verdict
none (the registered plant)	0.38	0.62	0.38	holds
loss scale 0.8	0.02	0.98	0.02	holds
constant-flow chilled water	0.12	0.88	0.12	holds
65 °F condenser-water floor	0.38	0.62	0.38	holds
twice the units at half the size	0.38	0.62	0.38	holds
least-power staging	0.38	0.62	0.38	holds
loss scale 1.2	0.76	0.24	0.24	holds
joint worst of 48: loss scale 0.8, constant-flow chilled water	-0.23	1.23	-0.23	flips

the nearest combination that holds does so by 0.02 kW. A more efficient plant lowers the evening chiller power by more than the daytime chiller power, so the campus peak falls toward the daytime plateau and the reduction the battery can achieve shrinks; constant-flow chilled water, colder at part load, compounds it. The staging rule, the unit count and the 65 °F floor leave both clauses unchanged at these duties.

5 Discussion

The result concerns one fixed schedule, not the best one. The charge and discharge powers and hours were chosen in advance, and the report checks their consequences; an optimized dispatch in the manner of Oudalov et al. (2007) would choose them against a tariff and could trade a slightly smaller discharge for a larger floor margin. The results bound what such a choice could gain here: with the campus peak 24.38 kW above the highest daytime hour, the 25 kW discharge already exceeds what the evening can use by 0.62 kW, so a discharge of 24.4 kW would cut the billed peak as much and leave more energy above the floor, where only 1.81 kWh remains.

The load is one synthetic day with flat hourly blocks. The peak is a plateau of 8 equal hours because the load is constant within the evening block and the discharge constant within the window; which hour is reported as the peak is therefore set by the schedule and the load shape together, and a measured load that varied within the evening would put the peak in one hour and make the reduction smaller than the rating wherever the load exceeded the evening average. Every hour is a settled steady state: transients of the racks and aisles within the hour and sub-hourly peaks that a meter with shorter intervals would bill are not represented. The battery's rated power limits are not enforced, and its heat goes to the ambient air rather than into the campus's cooling.

The chiller plant has no weather: its condenser water stays at the 85 °F rating temperature all day, while the IPLV that anchors the units credits part-load operation at 75 and 65 °F condenser water, so the plant runs at a condition less favorable than the one its efficiency level was set at. The chilled water is warmer than the rating point, 61 °F entering against 54 °F, which the law credits with a higher efficiency. The shape of the Gordon–Ng law is extrapolated from one centrifugal chiller to this class of positive-displacement units, and staged unloading of the compressors and short cycling are not represented beyond the time-averaged cycling branch (26). The site power

counts only the compressors, so the PUE is a lower bound on a real facility's. The sensitivity table shows these plant choices matter at the margin: the evening is absorbed under every plant, but a more efficient plant narrows the gap between the evening and the daytime and with it the reduction.

An earlier version of this study used hand-set conductances ($G_{rk} = 35$, $G_s = G_r = 80$, $G_{\mathrm{rm}} = 1,000$ and $G_b = 5,000$ W/K), a 290 K boundary uncoupled from a plant priced at 54 °F, and rack nodes up to 562 K. It reported a reduction of 25.00 kW with the peak held in the evening window over the hours ending 17–21 (88.56 to 63.56 kW). That result rested on the overheated racks: their leakage inflated the evening load more than the daytime load and kept the evening above the day even after the discharge. With conductances sized to the racks' heat the leakage is small, the evening falls below the daytime plateau, and the last 0.62 kW of the discharge is wasted; the earlier claim that the reduction did not depend on the network's temperatures was wrong.

6 Conclusion

This report asked whether a 220 kWh battery that charges at 5 kW for 6 night hours and discharges at 25 kW for 5 evening hours reduces the highest hourly grid import of a two-building data-center campus. It does, by 24.38 kW, from 83.81 kW to 59.43 kW, less than the 25 kW rating: after the discharge the daytime plateau sets the bill, so a discharge of about 24.4 kW would achieve the same cut and leave more energy above the charge floor. It asked whether the battery stays inside its limits. It does, narrowly on charge: it ends the day at a state of charge of 0.158 against a floor of 0.15, 1.8 kWh above it, and it peaks at 310.3 K against a 318 K limit, converting 15.4 kWh into heat; the charge floor, not the temperature, is the binding limit. It asked whether the reduction depends on how the chiller plant is modeled. Partly: the evening is absorbed under all 48 combinations of loss level, staging, unit count, condenser floor and chilled-water model, but in the 8 that pair a 20 % more efficient plant with constant-flow chilled water less than 24 kW of the schedule lands on the peak. These answers hold for one fixed schedule on one synthetic day of settled hourly operating points with a plant that sees no weather. The next step is to choose the schedule by optimization against a measured load profile and a tariff, with the site meter's and the battery's readings entering the same model.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level0_site/bess_peak_shaving
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
```

`run.py` writes `results.json`; `make_report.py` reads only `results.json` and builds the PDF with pdflatex. The computation is deterministic and uses no random numbers. The committed results were generated with Python 3.12; every number printed in this report is the same under the OpenBLAS kernels the gate tests exercise.

B Data and problems

The campus, its load profile, the battery, its schedule and the ambient temperature are the study's own construction, stated in Sections 2 and 3; no external or measured data are used. The chiller law's reference coefficients are fixed in the code of the chiller-plant model and stated in Section 3. The study leaves no corpus problems.

C Hourly operating points

Table 2 lists the operating point of every hour of the day.

Table 2: The day hour by hour. IT power with rack leakage, chiller compressor power and its coefficient of performance (heat removed per unit of compressor power), site power, battery net output (positive when discharging), grid import, and the battery's state of charge and temperature at the end of the hour.

hour ending	P_{IT} [kW]	P_{ch} [kW]	COP	P_{site} [kW]	P_{net} [kW]	P_{grid} [kW]	SOC	T_B [K]
1	42.25	8.13	5.20	50.38	−5.00	55.38	0.682	299.0
2	42.25	8.13	5.20	50.38	−5.00	55.38	0.703	299.0
3	42.25	8.13	5.20	50.38	−5.00	55.38	0.725	299.0
4	42.25	8.13	5.20	50.38	−5.00	55.38	0.746	299.0
5	42.25	8.13	5.20	50.38	−5.00	55.38	0.768	299.0
6	42.25	8.13	5.20	50.38	−5.00	55.38	0.790	299.0
7	48.03	8.73	5.50	56.76	0.00	56.76	0.790	298.9
8	48.03	8.73	5.50	56.76	0.00	56.76	0.790	298.9
9	50.44	8.99	5.61	59.43	0.00	59.43	0.790	298.9
10	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.0
11	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.1
12	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.2
13	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.4
14	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.6
15	50.44	8.99	5.61	59.43	0.00	59.43	0.790	299.8
16	50.44	8.99	5.61	59.43	0.00	59.43	0.790	300.0
17	72.12	11.70	6.17	83.81	25.00	58.81	0.663	302.5
18	72.12	11.70	6.17	83.81	25.00	58.81	0.537	304.8
19	72.12	11.70	6.17	83.81	25.00	58.81	0.411	306.8
20	72.12	11.70	6.17	83.81	25.00	58.81	0.284	308.7
21	72.12	11.70	6.17	83.81	25.00	58.81	0.158	310.3
22	48.03	8.73	5.50	56.76	0.00	56.76	0.158	309.4
23	48.03	8.73	5.50	56.76	0.00	56.76	0.158	308.5
24	48.03	8.73	5.50	56.76	0.00	56.76	0.158	307.6

D Notation

symbol	meaning	unit
P_i, P_0	rack electrical power and its base value	W
α, T_{ref}	rack leakage coefficient and reference temperature	W/K, K
T_i, T_c, T_h, T_{rm}, T_b	rack, cold-aisle, hot-aisle, room and building node temperatures	K
Q_i, Q_{sup}, Q_{mix}, Q_{ret}, Q_{rm}, Q_b	heat flows of the thermal network	W
G_{rk}, G_s, G_r, G_{rm}, G_b, λ	thermal conductances and the mixing factor	W/K, –
T_{chw}	chilled-water boundary of the network, the chiller's entering water	K
Q, C_u, n, k	plant duty, unit capacity, installed and running units	W, W, –, –
P_{GN}, b_1, b_2, b_3	Gordon–Ng unit power and its three loss coefficients	W, W/K, W, K/W
T_{chi}, T_{ci}	chilled water and condenser water entering the chiller	K
s, $q_{\min}$	loss factor of the anchored unit and its minimum stable load	–, W
E, SOC, C	stored energy, state of charge and capacity of the battery	J, –, J
P_c, P_d, P_{net}, P_{loss}	charge, discharge, net output and loss power of the battery	W
η_c, η_d	charge and discharge efficiencies	–
T_B, T_a, C_B, H	battery and ambient temperatures, battery heat capacity and loss conductance	K, K, J/K, W/K
P_{site}, P_{grid}, ΔP	site power, grid import and peak reduction	W
c_1, c_2, g	sensitivity clauses and margin	kW

References

[Oudalov et al. 2007] Oudalov, A., Cherkaoui, R., Beguin, A. (2007). Sizing and optimal operation of battery energy storage system for peak shaving application. In: 2007 IEEE Lausanne Power Tech, pp. 621–625. doi:10.1109/PCT.2007.4538388.

[Chua et al. 1996] Chua, H. T., Ng, K. C., Gordon, J. M. (1996). Experimental study of the fundamental properties of reciprocating chillers and their relation to thermodynamic modeling and chiller design. International Journal of Heat and Mass Transfer 39(11), 2195–2204. doi:10.1016/0017-9310(95)00335-5.

[Barroso et al. 2019] Barroso, L. A., Hölzle, U., Ranganathan, P. (2019). The Datacenter as a Computer: Designing Warehouse-Scale Machines. Synthesis Lectures on Computer Architecture, Springer. doi:10.1007/978-3-031-01761-2.

[ASHRAE 2019] ASHRAE (2019). ANSI/ASHRAE/IES Standard 90.1-2019: Energy Standard for Buildings Except Low-Rise Residential Buildings. ASHRAE, Atlanta, GA.