Hour-by-hour versus annual-average pricing of a campus chiller plant with free cooling and thermal storage

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Abstract

Cooling a data center campus draws a large share of its electricity, and annual budgets for that electricity are often set from a single efficiency number. This report tests such a number against an hour-by-hour model of a campus chiller plant through a synthetic year of 8,760 hours, in which the cooling load ranges from 430 to 816 kW and the outdoor wet bulb from about 278 to 298 K. A free-cooling path, a 2,400 kWh cold-water tank on a fixed rule and four staged 250 kW water-cooled chillers serve the load; each chiller is the least efficient that meets the ASHRAE 90.1 minimums for its size, and each hour is priced with the Gordon–Ng chiller law at the condenser water a cooling tower returns. The question is whether a dashboard efficiency (coefficient of performance, COP) of 8.1, set by hand, prices the year like the hourly model, and whether the difference crosses a 100 MWh budget line. The hourly model gives 682.3 MWh against the dashboard's 588.7 MWh: the dashboard misses 93.7 MWh, 13.7 % of the hourly-model energy, and stays under the line by 6.3 MWh, because the plant averages a COP of 6.99 over the year, not 8.1. The verdict is fragile. Of 36 combinations of four plant-modeling choices, three single changes push the gap over the line (constant-flow chilled water, condenser-water floor 65 °F and losses scaled by 1.2), and the worst combination misses 303 MWh. Resolving the year changes the energy forecast by about 14 %; whether that crosses a budget line depends on plant details a dashboard does not record. The study is a forward one: every input is stated and nothing is estimated from data. It is closed-form evaluation throughout, 8,760 hours for each of the 36 plant variants.

1 Introduction

A data center turns nearly all the electricity its servers draw into heat, and removing that heat is its largest energy use after the computing itself. Barroso, Hölzle and Ranganathan (2019) treat the cooling plant as part of the warehouse-scale computer and its efficiency as a first-order design quantity, and Ebrahimi, Jones and Fleischer (2014) review the cooling technologies and operating conditions in use. The electricity a cooling plant draws depends on the hour: a chiller's efficiency changes with its load and with the temperature of the condenser water a cooling tower can deliver, which follows the outdoor wet bulb; free cooling carries part of the load in cold weather and none in warm weather; thermal storage moves load from one hour to another. Zhang, Shao, Xu, Zou and Tian (2014) review how much free cooling saves across climates and system types. Budgets, capacity plans and utility contracts nevertheless often price a year with one number, an annual efficiency divided into the annual cooling load, because it is quick and fits a dashboard. Whether that number is accurate enough to support a budget decision is the question this report takes up on one plant.

Two families of methods sit on either side of the flat number. Building energy simulation resolves the year hour by hour with component models of chillers, cooling towers and storage; it is typically run once, at design time. Thermodynamic chiller models make such a resolution cheap and physically grounded: the Gordon–Ng family writes a chiller's efficiency from the first and second laws with three loss coefficients, for internal entropy generation, heat leak and heat-exchanger resistance, and Chua, Ng and Gordon (1996) examine experimentally the properties such models rest on. The efficiency a new chiller must reach is fixed by ASHRAE Standard 90.1 (ASHRAE, 2019) at the AHRI 550/590 rating conditions, both at full load and as an integrated part-load value. None of these constructions is new here; the report applies them to one question.

TERRA is a general-purpose engine for probabilistic inference on physical networks. It treats a network as a graph on which the physics is written locally: a conservation law at every node, a constitutive law on every edge, and the boundary and component laws that close them, attached to the same graph. Each of these is a relation the state of the network must satisfy, and the engine assembles them and solves them together. Domains enter through plugins that supply their laws, units and file formats; the solving and the bookkeeping are shared. This study uses the heating, ventilation and air-conditioning plugin's chiller law, its anchoring to the code minimums and its staged-plant construct to price a campus cooling plant hour by hour through a year; it evaluates the plant's relations directly and estimates nothing from readings. Section 3 states the framework and each of these constructs precisely.

This report asks whether one flat annual efficiency prices a campus plant's year like an hour-by-hour model of the same plant, and whether the difference crosses a 100 MWh budget line. It builds the hourly model from a free-cooling limit, a storage rule, sticky chiller staging and a code-minimum chiller law, compares it with a dashboard's flat efficiency on identical loads, and tests the budget verdict across 36 combinations of plant-modeling choices. It is a forward study: every input is stated and nothing is estimated from data. Section 2 states the plant and the synthetic year, Section 3 the model, the baseline and the protocol, Section 4 the results, Section 5 their limits and Section 6 the answers.

2 System and data

The plant. Figure 1 draws the plant. A data center campus produces heat at a rate that changes with the hour, and a chilled-water loop carries it to the cooling plant. Three paths remove it. A free-cooling heat exchanger (a water-side economizer) passes heat to the cooling-tower water without a compressor whenever the outdoor air is cold enough. Four identical 250 kW water-cooled positive-displacement chillers, vapor-compression machines with screw or scroll compressors, are staged on and off as the load moves. A 2,400 kWh cold-water tank stores cooling: it is charged by the chillers in cool, light hours and discharged into the load in hot, heavy ones. The chillers reject their heat to condenser water returned by a cooling tower. The tower cannot cool its water below the outdoor wet bulb, the temperature air reaches when water evaporates into it, and the margin by which the water stays above the wet bulb, the tower's approach, is held at its design value of 7 °F (3.89 K). Head-pressure control never lets the condenser water fall below 55 °F (285.93 K). Chilled water enters each chiller at 54 °F (285.37 K) and leaves at 44 °F, the AHRI 550/590 rating temperatures. A chiller's efficiency is its coefficient of performance (COP), the heat it removes divided by the electricity its compressor draws. Each unit is the least efficient 250 kW water-cooled positive-displacement chiller that meets both ASHRAE 90.1 minimums for its size: its full-load COP at the rating conditions is 5.58, against a code minimum of 4.69, because the code's part-load minimum is the stricter of the two. The quantity of interest is the compressor electricity of the

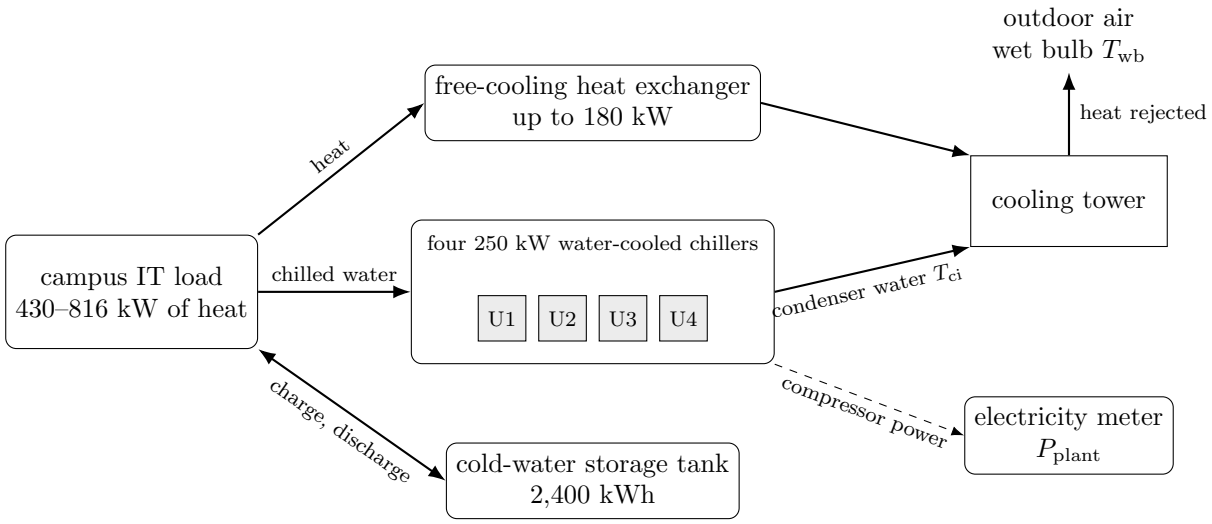


Figure 1: The campus cooling plant. Heat from the IT load reaches the plant in chilled water and leaves by one of three paths: a free-cooling heat exchanger that rejects it through the cooling tower without a compressor when the outdoor wet bulb is low, four identical water-cooled chillers staged on and off, and a cold-water tank that is charged by the chillers and discharged into the load. The chillers reject their heat to condenser water from the cooling tower, whose temperature follows the outdoor wet bulb. The meter records the chillers' compressor power, the quantity this report prices.

plant over a year; pumps, tower fans and pipes are not priced.

The data. No measured data are used. The year is synthetic and deterministic: 8,760 hourly values of the cooling load and the outdoor wet bulb, from a seasonal wave s_h, a daytime wave d_h and a business-hours step b_h, for hours $h = 0, \ldots, 8759$ of a 365-day year, counted from midnight on January 1,

$$Q_{\mathrm{load},h} = \max\big(360,\ 520 + 165\,d_h + 90\,s_h + 42\,b_h\big), \tag{1}$$

$$T_{\mathrm{wb},h} = 286 + 8\,s_h + 4.5\,d_h, \tag{2}$$

$$s_h = \sin\big(2\pi\,(\lfloor h/24 \rfloor - 172)/365\big), \tag{3}$$

$$d_h = \max\big(0,\ \sin\big(\pi\,(h \bmod 24 - 6)/15\big)\big), \tag{4}$$

$$b_h = \begin{cases} 1 & \text{if } 8 \le h \bmod 24 \le 19, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

with the load in kW and the wet bulb in K. The seasonal wave is highest on day 264 of the year (September 21) and lowest in late March; the daytime wave is positive from 6:00 for 15 hours. The load ranges from 430.0 to 816.1 kW, so the 360 kW lower bound in (1) never acts, and the wet bulb from 278.0 to 298.5 K. Nothing is random, and no noise is added.

The model. Figure 2 shows how one hour is evaluated and how the year is assembled. The inputs of an hour are its load and wet bulb. Free cooling takes the share the weather allows, the tank rule decides whether the tank charges, discharges or holds, and the remainder is the load on the chillers. The staging rule sets how many units run, the units share the load equally, and the chiller

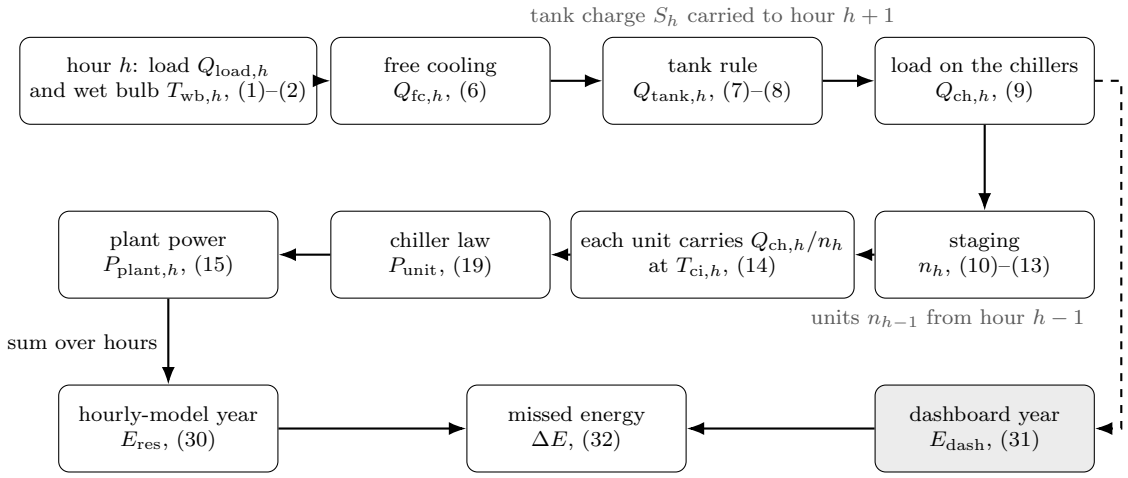


Figure 2: The model of one hour and of the year. Solid arrows: the hourly model, evaluated in order for $h = 0, \ldots, 8759$; each box names the quantity it produces and the equation that defines it. Two quantities carry state from one hour to the next: the tank's charge and the number of running units. The dashed arrow is the dashboard baseline, which prices the same load on the chillers at one flat efficiency. The difference of the two annual sums is the energy the dashboard misses.

law gives each unit's power at its share and at the condenser water the tower returns. The tank's charge and the number of running units carry over to the next hour; nothing else does. Section 3 states each relation with its constants.

Assumptions. The model rests on the following choices.

- Units: kW for power and heat flow, kWh and MWh for energy, K for temperature (degrees Fahrenheit where a rating or code defines the value), one-hour steps.
- Each hour is a settled operating point: no start-up, no transients, no short cycling.
- Free cooling is subtracted from the load; the chilled water entering the chillers stays at the 54 °F rating temperature at every load.
- The tower approach is fixed at 3.89 K and the condenser-water floor at 55 °F.
- The tank follows a fixed rule, not an optimized dispatch, and has no thermal losses.
- Pumps, tower fans and distribution pipes are not priced.
- The dashboard's COP of 8.1 is set by hand and is not derived from the plant.
- Every input is stated and every relation is evaluated: there are no readings, no noise and nothing estimated from data.

The questions. (1) Does a flat COP of 8.1 price the year like the hourly model, and by how much do they differ? (2) Does the difference cross the 100 MWh budget line? (3) Does the answer to (2) hold under alternative, defensible choices in how the chiller plant is modeled?

3 Methods

3.1 The hourly plant model

Every hour of the year is evaluated by the same relations, in the order of Figure 2; Table 4 in Appendix C collects the symbols. Free cooling carries at most 180 kW, at most 18 % of the load, and 24 kW for every kelvin the wet bulb lies below 293 K, so that it fades to nothing at that wet bulb. The tank discharges 120 kW into the load in hot, heavy hours while it holds enough charge, and is charged at 70 kW in cool, light hours while it has room; its charge S_h is a fraction of its 2,400 kWh capacity and starts the year at 0.65. The chillers carry what remains, never less than 1 kW. Staging is sticky: with $C = 250$ kW the rated capacity of one unit, a count of k running units is admissible when the load lies between L_k and U_k, with $L_1 = -\infty$ and $U_4 = +\infty$; the previous hour's count is kept while it stays admissible, and otherwise the smallest admissible count is taken. A unit therefore starts when the running units would exceed 90 % of their capacity and stops when the units left would carry the load at no more than 68 % of theirs. The year starts with one unit running. The condenser water entering the chillers is the wet bulb plus the tower approach ΔT_{a} of 3.89 K, never below the floor T_{floor} of 285.93 K, and each running unit carries an equal share of the load:

$$Q_{\mathrm{fc},h} = \min\big(180,\ 0.18\,Q_{\mathrm{load},h},\ 24\,\max(0,\ 293 - T_{\mathrm{wb},h})\big), \tag{6}$$

$$Q_{\mathrm{tank},h} = \begin{cases} 120 & \text{if } Q_{\mathrm{load},h} > 720,\ T_{\mathrm{wb},h} > 294,\ S_h > 0.25, \\ -70 & \text{if } Q_{\mathrm{load},h} < 520,\ T_{\mathrm{wb},h} < 290,\ S_h < 0.90, \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

$$S_{h+1} = \begin{cases} \max\big(0.18,\ S_h - 120/2400\big) & \text{when discharging,} \\ \min\big(0.92,\ S_h + 70/2400\big) & \text{when charging,} \\ S_h & \text{when holding,} \end{cases} \tag{8}$$

$$Q_{\mathrm{ch},h} = \max\big(1,\ Q_{\mathrm{load},h} - Q_{\mathrm{fc},h} - Q_{\mathrm{tank},h}\big), \tag{9}$$

$$\mathcal{A}(Q) = \{\, k \in \{1,\dots,4\} : L_k \le Q \le U_k \,\}, \tag{10}$$

$$L_k = 0.68\,(k-1)\,C, \tag{11}$$

$$U_k = 0.90\,k\,C, \tag{12}$$

$$n_h = \begin{cases} n_{h-1} & \text{if } n_{h-1} \in \mathcal{A}(Q_{\mathrm{ch},h}), \\ \min \mathcal{A}(Q_{\mathrm{ch},h}) & \text{otherwise,} \end{cases} \tag{13}$$

$$T_{\mathrm{ci},h} = \max\big(T_{\mathrm{wb},h} + \Delta T_{\mathrm{a}},\ T_{\mathrm{floor}}\big), \tag{14}$$

$$P_{\mathrm{plant},h} = n_h\,P_{\mathrm{unit}}\big(Q_{\mathrm{ch},h}/n_h,\ T_{\mathrm{ci},h}\big). \tag{15}$$

Relations (6)–(14) are closed-form rules, and (15) prices the hour with the chiller law of Section 3.3. The tank's charge (8) and the running units (13) are the only quantities one hour passes to the next.

3.2 The network model

The relations of an hour are one instance of a general construction, in which the unknowns of a network and the relations they must satisfy are written on its graph and solved together; this subsection states it so that the constructs above have a precise place. The energy balance of an hour, (9), is a conservation relation, and the chiller law (19) is a constitutive relation with an

analytic gradient; here they are chained in one direction, so the closed forms above give the hour's state directly, without an iteration. A physical network is a graph, and the engine writes its physics on that graph. The state is a vector z of unknowns: one potential per node, such as a temperature or a head, one flow per edge, and whatever else the physics determines. Every relation the state must satisfy — a conservation law at a node, a constitutive law on an edge, a boundary condition, a component law — is written as a residual $r_k(z)$ that vanishes when the relation holds. With as many independent relations as unknowns and no readings to reconcile, the state is the unique solution of

$$r(z) = 0, \tag{16}$$

found by Newton iteration on the sparse Jacobian J of the residuals, $J_{ki} = \partial r_k / \partial z_i$, from a stated start until the residual norm falls below a tolerance ϵ:

$$z \leftarrow z - J^{-1} r(z) \quad \text{until } \|r(z)\| < \epsilon. \tag{17}$$

A transient model adds a storage term to the conservation relations and advances the state in time with an implicit integrator, solving a system of the same kind at every step. Nothing in this report is estimated from data: every input is stated, and the results are the solutions of these relations and quantities computed from them. Everything that follows is a choice of unknowns and relations on this graph.

3.3 The chiller law and its anchoring

Each unit's compressor power follows the Gordon–Ng universal chiller law, which applies the first and second laws of thermodynamics to the refrigeration cycle and describes the departure from the reversible (Carnot) machine with three loss coefficients: b_1, the internal entropy generation [W/K]; b_2, an equivalent heat leak [W]; and b_3, the total heat-exchanger resistance [K/W]. With Q the unit's load, T_{chi} the chilled water entering its evaporator and T_{ci} the condenser water entering its condenser, the law and its solution for the compressor power are

$$\frac{T_{\mathrm{chi}}}{T_{\mathrm{ci}}}\Big(1 + \frac{1}{\mathrm{COP}}\Big) = 1 + \frac{b_1 T_{\mathrm{chi}}}{Q} + \frac{b_2 (T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{ci}}\, Q} + b_3 \Big(1 + \frac{1}{\mathrm{COP}}\Big)\frac{Q}{T_{\mathrm{ci}}}, \tag{18}$$

$$P_{\mathrm{unit}}(Q, T_{\mathrm{ci}}) = \frac{T_{\mathrm{ci}}\, Q + b_1 T_{\mathrm{chi}} T_{\mathrm{ci}} + b_2 (T_{\mathrm{ci}} - T_{\mathrm{chi}})}{T_{\mathrm{chi}} - b_3 Q} - Q, \tag{19}$$

$$P_{\mathrm{unit}}(Q, T_{\mathrm{ci}}) = \frac{Q}{Q_{\min}}\, P_{\mathrm{unit}}(Q_{\min}, T_{\mathrm{ci}}), \tag{20}$$

where $\mathrm{COP} = Q / P_{\mathrm{unit}}$. With $b_1 = b_2 = b_3 = 0$, (19) is the Carnot limit; the b_3 term gives the law an efficiency maximum at part load. Below a minimum stable load $Q_{\min}$, 10 % of the rated capacity, a real machine cycles on and off, and the time-averaged power (20) is linear in load. In this study the unit load never falls below 117.5 kW, so (20) never acts. Chilled water enters at $T_{\mathrm{chi}} = 285.37\,\mathrm{K}$ in every hour of the study's plant.

The shape of the law, the ratios between the three coefficients, is taken from a calibration on the 316.5 kW (90-ton) centrifugal chiller of the ASHRAE RP-1043 laboratory tests, whose coefficients b_1^{ref}, b_2^{ref} and b_3^{ref} are $78.43\,\mathrm{W/K}$, $1.220 \times 10^5\,\mathrm{W}$ and $7.711 \times 10^{-5}\,\mathrm{K/W}$. A unit of capacity C scales the entropy generation and the heat leak with capacity and the heat-exchanger resistance inversely

with it, and one loss factor f sets the efficiency level:

$$b_1 = f\, b_1^{\mathrm{ref}}\, C/C_{\mathrm{ref}}, \tag{21}$$

$$b_2 = f\, b_2^{\mathrm{ref}}\, C/C_{\mathrm{ref}}, \tag{22}$$

$$b_3 = f\, b_3^{\mathrm{ref}}\, C_{\mathrm{ref}}/C, \tag{23}$$

$$T_{\mathrm{chi}}^{(x)} = T_{\mathrm{chw,l}} + \Delta T_{\mathrm{d}}\, x, \tag{24}$$

$$\mathrm{COP}_x = \frac{x\, C}{P_{\mathrm{unit}}\big(x\, C,\ T_{\mathrm{ci}}^{(x)}\big)}, \tag{25}$$

$$\mathrm{IPLV} = 0.01\,\mathrm{COP}_1 + 0.42\,\mathrm{COP}_{0.75} + 0.45\,\mathrm{COP}_{0.5} + 0.12\,\mathrm{COP}_{0.25}, \tag{26}$$

$$\mathrm{COP}_1(f_{\mathrm{FL}}) = \mathrm{COP}_{\mathrm{FL}}^{\mathrm{min}}, \tag{27}$$

$$\mathrm{IPLV}(f_{\mathrm{IPLV}}) = \mathrm{COP}_{\mathrm{IPLV}}^{\mathrm{min}}, \tag{28}$$

$$f = \min(f_{\mathrm{FL}},\ f_{\mathrm{IPLV}}). \tag{29}$$

Here C_{ref} is 316.5 kW, and COP_x is the unit's COP at the load fraction x under the AHRI 550/590 water-cooled rating: chilled water leaves at $T_{\mathrm{chw,l}} = 44\,^{\circ}\mathrm{F}$ with a full-load rise $\Delta T_{\mathrm{d}} = 10\,^{\circ}\mathrm{F}$ under constant evaporator flow (24), and condenser water enters at 85, 75, 65, 65 °F at 100, 75, 50 and 25 % load, with the 55 °F floor applied. The integrated part-load value (26) is the AHRI-weighted mean of the four COPs. ASHRAE 90.1-2019, Table 6.8.1-3, Path A, requires a water-cooled positive-displacement chiller below 75 tons, such as this 71.1-ton unit, to reach 0.750 kW/ton at full load (COP 4.69) and 0.600 kW/ton as IPLV (COP 5.86), and both requirements must be met. Each requirement fixes the factor at which the unit just meets it, (27) and (28), found by bisection because the COP falls monotonically as f grows; the least efficient unit meeting both takes the smaller factor (29). Here the IPLV requirement binds: the loss factor is 0.597, the unit's IPLV is 5.86 and its full-load COP 5.58, and its coefficients b_1, b_2 and b_3 are 37.0 W/K, 5.756×10^4 W and 5.833×10^{-5} K/W. The calibration covered chilled water entering at 279.0–289.6 K, condenser water at 288.6–303.2 K, loads of 24–118 % of capacity and lifts (condenser water minus chilled water) of 4.9–20.8 K; outside that range the law extrapolates.

3.4 Annual energy and the dashboard baseline

The hourly model's annual energy is the sum of the plant power over the year. The dashboard prices the same hourly load on the chillers, after free cooling and the tank, at one flat COP, $\mathrm{COP}_{\mathrm{dash}} = 8.1$, a hand-set number not derived from the plant; the two therefore differ only in efficiency, never in load. The missed energy is their difference, and the plant's annual average COP is the cooling it delivers over the electricity it draws:

$$E_{\mathrm{res}} = \sum_{h=0}^{8759} P_{\mathrm{plant},h} \cdot 1\,\mathrm{h}, \tag{30}$$

$$E_{\mathrm{dash}} = \sum_{h=0}^{8759} \frac{Q_{\mathrm{ch},h}}{\mathrm{COP}_{\mathrm{dash}}} \cdot 1\,\mathrm{h}, \tag{31}$$

$$\Delta E = E_{\mathrm{res}} - E_{\mathrm{dash}}, \tag{32}$$

$$\overline{\mathrm{COP}} = \frac{\sum_h Q_{\mathrm{ch},h}}{\sum_h P_{\mathrm{plant},h}}. \tag{33}$$

Because $E_{\mathrm{dash}} = E_{\mathrm{res}}\,\overline{\mathrm{COP}}/\mathrm{COP}_{\mathrm{dash}}$, the dashboard understates the year exactly when the plant's average COP (33) falls short of 8.1.

3.5 Sensitivity of the budget verdict to the plant modeling choices

The verdict, whether ΔE exceeds 100 MWh, rests on choices in how the chiller plant is modeled that a reader could reasonably make differently, so it is put through a sensitivity analysis over those choices. Four of them are varied. The loss scale multiplies the anchored coefficients b_1, b_2, b_3 by 0.8 or 1 or 1.2. The staging is either the study's sticky staging of four 250 kW units, or two 500 kW units, each anchored again at its own size (142.2 tons, in the 75–150 ton class: 0.720 kW/ton at full load and 0.560 kW/ton as IPLV), staged either by threshold, running $\min\left(2, \lceil Q_{\mathrm{ch}}/(0.90 \cdot 500\,\mathrm{kW})\rceil\right)$ units, or for least power, running whichever of one or two units draws less while carrying the load, ties going to one. The condenser-water floor is 55 °F or 65 °F, the unit being anchored again with that floor applied at the rating points. The chilled water entering the unit is either fixed at 54 °F or falls with the unit's load fraction under constant evaporator flow, (24) with $x = Q/C$. All 36 combinations are priced over the same hourly loads and compared with the same E_{dash}, and each is scored by its margin

$$g = \Delta E - 100\,\mathrm{MWh}, \tag{34}$$

positive when the gap is over the line. The study's plant is the combination of losses as anchored, sticky staging, the 55 °F floor and fixed chilled water. A combination flips the verdict when its margin has the other sign. The report lists the study's plant, every combination that changes one choice, and the joint worst combination, the one whose margin lies furthest toward overturning the study's verdict, ties going to the earlier levels in the order listed above.

3.6 Metrics

The first question is answered by the annual energies E_{res} and E_{dash} in MWh and the missed energy ΔE of (32), also as a share of E_{res} in percent; the reference is zero, which a dashboard that priced the year correctly would give. The annual average COP (33) explains the sign and size of ΔE and is read against the dashboard's 8.1. Monthly energies and the COP over wet-bulb bins show where in the year the difference arises, and the highest hourly draw in kW, with the plant's regime in that hour, shows what an annual number cannot represent. The census counts hours out of 8,760: running units, tank modes, free-cooling hours, hours at the condenser-water floor and hours in which a unit operates outside the chiller law's calibrated range, the last also weighted by the plant energy in those hours; it shows which regimes carry the year and how much of the result rests on extrapolation. The second question is answered by the margin g of (34) in MWh for the study's plant, read against zero. The third is answered by the number of single changes, of six, whose margin changes sign, and by the missed energy of the joint worst combination, read against the 100 MWh line. These margins are a sensitivity analysis over modeling choices: each combination is a different plant, priced exactly, and the spread of the margins says how far the verdict depends on which plant a reader would build.

4 Results

Annual energy. Table 1 compares the two ways of pricing the year: the hourly model in the first column of numbers, the dashboard in the second, and their difference in the third. The hourly model gives 682.3 MWh of chiller electricity and the dashboard 588.7 MWh for the same hourly loads, so the dashboard misses $\Delta E = 93.7$ MWh, 13.7 % of the hourly-model energy, against a reference of zero. The sign and size follow from the average COP: the plant delivers its 4,768 MWh of cooling at an average COP of 6.99, and the dashboard assumes 8.1, so by (33) it prices the year

13.7 % too low. The missed energy lies 6.3 MWh under the 100 MWh budget line: the dashboard's error is real, but on the study's plant it does not change the budget decision. In the worst hour the plant draws 147.2 kW, 46.5 kW more than the dashboard's highest hourly figure.

Table 1: The synthetic year priced two ways. Columns: the hourly model, the dashboard's flat COP applied to the same hourly load on the chillers, and their difference (hourly model minus dashboard). The margin is the missed energy minus the 100 MWh budget line; a negative margin keeps the gap under the line.

quantity	hourly model	dashboard	difference
annual chiller electricity [MWh]	682.3	588.7	93.7
missed share of the hourly-model energy [%]			13.7
annual average COP	6.99	8.1	–
highest hourly draw [kW]	147.2	100.8	46.5
margin to the 100 MWh line [MWh]			−6.3

Energy by month. Figure 3 splits the year by calendar month, with a dark bar for the hourly model and a light bar for the dashboard in each month. The dashboard overstates the energy from January to May, by 14.4 MWh in total, and understates it from June to December, by 108.1 MWh; the net is the 93.7 MWh of Table 1. The largest monthly shortfall is in September, 86.6 MWh against 61.8 MWh. A flat COP therefore does not merely scale the year down: it moves energy from the warm months, where the plant is least efficient, into the cool ones.

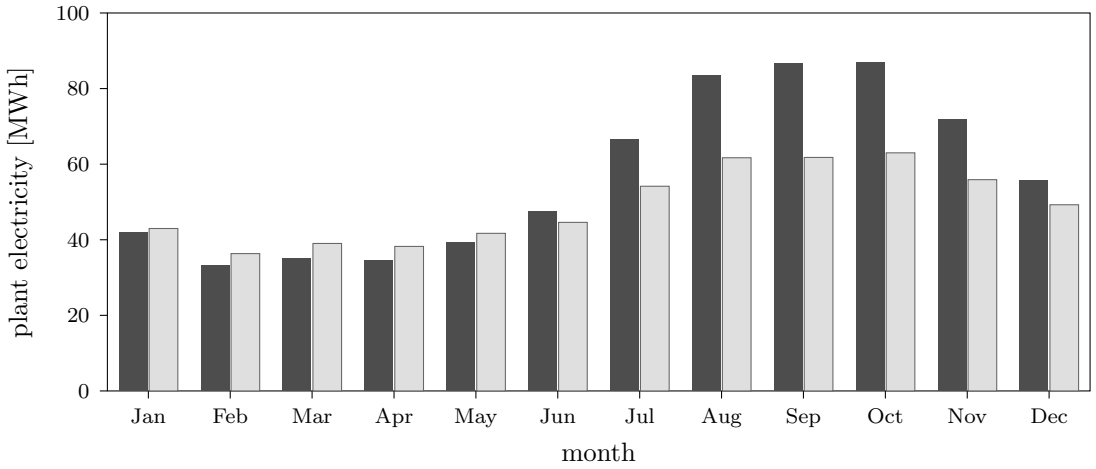
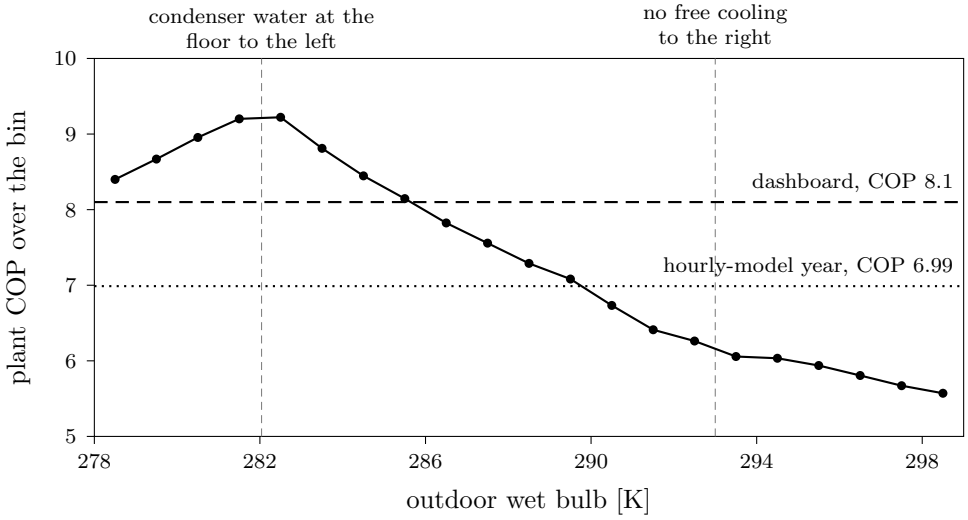


Figure 3: Chiller electricity per calendar month of the synthetic year, from the hourly model (dark) and from the dashboard's flat COP of 8.1 applied to the same load (light). The dashboard overstates January to May and understates every other month; the understatement dominates the year.

Efficiency and weather. Figure 4 explains the monthly pattern. It plots, for each 1 K bin of outdoor wet bulb, the cooling the chillers delivered in the hours of that bin divided by the electricity they drew, with the dashboard's COP dashed and the hourly model's annual average dotted. The plant's COP exceeds 8.1 only in the bins below 286 K, which hold 3,728 of the 8,760 hours, and

reaches its highest value, 9.22, in the 282–283 K bin, next to the wet bulb of 282.0 K below which the 55 °F floor holds the condenser water fixed. Above that wet bulb each kelvin of warmer weather raises the condenser water (14) and lowers the COP; below it the condenser water no longer falls, and the COP declines with the lighter loads of those hours. In the warmest bins, above the 293 K at which free cooling stops, the COP is 5.57 to 6.06, and the warm half of the year carries the missed energy.



dots: cooling delivered over electricity drawn, per 1 K wet-bulb bin; dashed: dashboard; dotted: annual average

Figure 4: The hourly model's plant COP against the outdoor wet bulb, with the hours of the year grouped into 1 K bins (142 to 852 hours per bin). The dashed line is the dashboard's flat COP and the dotted line the year's average from the hourly model. The vertical lines mark the wet bulb below which the 55 °F floor sets the condenser water and the wet bulb above which free cooling stops.

Operating regimes. Table 2 counts the hours of the year by regime. The plant ran three units for 5,401 hours and four units for 3,359 hours, and never one or two: three units run from the first hour, and after free cooling and the tank the chillers never carry less than 353 kW, above the $L_3 = 340$ kW of (11) below which the third unit would stop. The tank charged in 33 hours and discharged in 14, holding for the remaining 8,713; its rule asks for a hot, heavy hour and a charged tank together, and this smooth year rarely offers both. Free cooling carried load in 6,450 hours, 74 % of the year, and 545 MWh of the 5,312 MWh of cooling. The condenser water sat at its floor in 1,890 hours. In 3,868 hours, 44 % of the year and 28 % of the chiller energy, a unit ran outside the range its law was calibrated on, every one of them because the lift was smaller than the smallest measured, 4.9 K. The worst hour of the year is hour 6,326, September 21 at 14:00: the load is at its maximum of 816 kW, the wet bulb is 298.5 K, free cooling carries 0 kW, the tank holds because its charge is at or below the 25 % it needs to discharge, and all four units run at a COP of 5.54 to draw 147.2 kW. The dashboard prices that hour at 100.8 kW.

Sensitivity of the budget verdict. Table 3 prices the year again under the alternative plant-modeling choices of Section 3.5: the study's plant first, then each single change, then the joint worst of all 36 combinations, with the missed energy, the margin (34) and the side of the line for each. On

Table 2: Census of the 8,760 hours of the year: how many chillers ran, what the tank did, how often free cooling carried load (more than 1 kW), how often the condenser-water floor was reached, and how often a running unit operated outside the range its chiller law was calibrated on. Each block sums to the year where its rows are exclusive.

condition	hours of 8,760
1 chiller running	0
2 chillers running	0
3 chillers running	5,401
4 chillers running	3,359
tank charging	33
tank discharging	14
tank holding	8,713
free cooling active	6,450
condenser water at the 55 °F floor	1,890
unit operating point outside the chiller law's calibrated range	3,868

the study's plant the margin is −6.3 MWh, 0.9 % of the hourly-model energy. Three of the six single changes push the gap over the line: losses scaled by 1.2 (+108.2 MWh); constant-flow chilled water (+41.2 MWh); condenser-water floor 65 °F (+34.6 MWh). Scaling the losses down, or replacing the four sticky units by two 500 kW units under either staging rule, keeps it under. The joint worst combination, losses scaled by 1.2, condenser-water floor 65 °F, constant-flow chilled water, misses 303.0 MWh, 3.0 times the budget line. Resolving the year changes this plant's energy forecast by about 14 %; whether that crosses a budget line depends on the unit losses, the condenser-water control and the chilled-water flow, details a dashboard does not record.

Table 3: The budget verdict under alternative plant-modeling choices. Each row changes the listed choices from the study's plant (four 250 kW units with sticky staging, losses as anchored, a 55 °F condenser-water floor, chilled water entering at a fixed temperature) and prices the year again. Missed energy is the hourly-model energy minus the dashboard's; the margin is the missed energy minus the 100 MWh line; the last column says on which side of the line the gap falls. The last row is the combination, out of all 36, with the largest margin.

choice changed	missed energy [MWh]	margin [MWh]	gap versus line
none (the study's plant)	93.7	−6.3	under
losses scaled by 0.8	−19.3	−119.3	under
losses scaled by 1.2	208.2	+108.2	over
two 500 kW units, threshold staging	49.8	−50.2	under
two 500 kW units, least-power staging	40.8	−59.2	under
condenser-water floor 65 °F	134.6	+34.6	over
constant-flow chilled water	141.2	+41.2	over
joint worst of 36: the three flipping changes	303.0	+203.0	over

5 Discussion

The year is synthetic and smooth. Its load and wet bulb are sums of sine waves and a step, with no heat waves, no humid spells and no irregular weeks. A measured year has hot, humid spells in which the plant's COP is lowest and cold snaps in which it is highest, and the gap found here is conditional on the smooth year. Each hour is a settled operating point, so start-up, part-load unloading steps and the efficiency lost to short cycling are not represented, and the chilled water enters the chillers at the 54 °F rating temperature in every hour. The storage rule is fixed, not optimized, and fires in only 47 hours; a dispatch chosen to shift load into efficient hours would change the census and could change the energy. Pumps, tower fans and pipes are not priced, so the energies are compressor energies, not plant totals.

The condenser side is simplified. The tower's approach is fixed at 3.89 K, although a real tower's approach grows with its heat load and shrinks at high wet bulb, and the 55 °F floor sets the condenser water in 1,890 hours, so the cold part of the year is priced at a control setting rather than at the weather. Free cooling is subtracted from the load, but a series water-side economizer would also cool the chilled water entering the chillers, by up to 1 K; on this unit that raises the chiller power by 3.3 to 4.9 % in the 6,450 free-cooling hours and would narrow the 6.3 MWh margin. The constant-flow row of Table 3 moves the chilled water in the same direction and flips the verdict.

The chiller law carries the largest caveat. Its shape was measured on one centrifugal chiller and is applied here to a smaller positive-displacement unit, an extrapolation across machine classes, and its level is anchored to code minimums rather than to a product. 44 % of the hours, carrying 28 % of the chiller energy, lie outside the lifts the law was calibrated on, all at lifts below 4.9 K; in those hours the law extrapolates toward the reversible limit, and the coefficient of performance it gives there is the least supported number in the study. The sensitivity table varies the level of the losses, the staging, the floor and the chilled water, but not the shape of the law, the tower approach or the weather. What the study does not do is estimate anything: it has no readings, and its conclusions are conditional on the stated plant and year. The margin that decides the budget question, 6.3 MWh or 0.9 % of the energy, is smaller than several of these effects, which is why the report states the verdict together with the choices that overturn it.

6 Conclusion

This report priced a campus cooling plant, with free cooling, a cold-water tank and four staged 250 kW code-minimum water-cooled chillers behind a cooling tower, hour by hour through a synthetic year, and compared the result with a dashboard that applies one flat efficiency to the same load. It asked first whether a flat COP of 8.1 prices the year like the hourly model: it does not. The hourly model gives 682.3 MWh and the dashboard 588.7 MWh, so the dashboard misses 93.7 MWh, 13.7 % of the energy, because the plant averages a COP of 6.99; the dashboard overstates the cool months and understates the warm ones by more, and it cannot show the worst hour, 147.2 kW with all units running and no free cooling. It asked second whether that gap crosses a 100 MWh budget line: on the study's plant it does not, by 6.3 MWh. It asked third whether that answer holds under alternative modeling choices: it does not hold robustly, since three single changes of six push the gap over the line (constant-flow chilled water, condenser-water floor 65 °F and losses scaled by 1.2) and the worst of 36 combinations misses 303 MWh. Resolving the year changes the energy forecast by about 14 %; whether that matters for a budget depends on plant details a dashboard does not record. The next step is to repeat the comparison on a measured weather year with a cooling-tower model whose approach varies with load and wet bulb.

A Reproduction

```
cd docs/terra_graph_engine/case_studies/level0_site/annual_campus_sweep
PYTHONPATH=../../../../../src python run.py
PYTHONPATH=../../../../../src python make_report.py --pdf
```

`run.py` writes `results.json`; `make_report.py` reads only `results.json` and writes `report.tex`, and with `--pdf` builds `report.pdf`. The study draws no random numbers, so there are no seeds; it takes a few seconds. Sums over the 8,760 hours can differ in their last digits between numerical libraries; every number in the text is rounded far above that. The gate test is `test_annual_campus_sweep_case_study.py` under `tests/plugins/datacenter`.

B Data and problems

The year is the study's own construction from (1)–(5); no external data are used. The chiller law's reference coefficients come from a calibration on the ASHRAE RP-1043 chiller test data, fixed as constants in the chiller-performance module of the heating, ventilation and air-conditioning plugin; the code minimums are those of ASHRAE 90.1-2019, Table 6.8.1-3, Path A, and the rating conditions those of AHRI 550/590. The study leaves no corpus problems.

C Notation

Table 4 lists the symbols.

Table 4: Symbols used in the report, with units.

symbol	meaning
h	hour of the year, $0, \ldots, 8759$
$Q_{\mathrm{load},h}$	campus cooling load [kW]
$T_{\mathrm{wb},h}$	outdoor wet-bulb temperature [K]
$Q_{\mathrm{fc},h}$	load carried by free cooling [kW]
$Q_{\mathrm{tank},h}$, S_h	tank discharge (negative when charging) [kW], tank charge as a fraction of capacity
$Q_{\mathrm{ch},h}$	load on the chillers [kW]
n_h, C	running units, rated capacity of one unit [kW]
T_{ci}, T_{chi}	condenser water and chilled water entering the unit [K]
ΔT_{a}, T_{floor}	cooling-tower approach [K], lowest condenser water allowed [K]
b_1, b_2, b_3	chiller-law loss coefficients [W/K], [W], [K/W]
f	loss factor anchoring the unit to the code minimums
P_{unit}, $P_{\mathrm{plant},h}$	compressor power of one unit and of the plant [kW]
COP	coefficient of performance, heat removed over electricity drawn
E_{res}, E_{dash}, ΔE	annual energy of the hourly model and the dashboard, and their difference [MWh]
g	margin to the budget line [MWh]

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